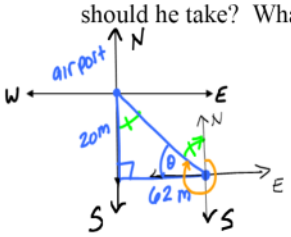
Thursday, January 17, 2019
3:09 PM

Precalculus
Test Review 4.5-4.8 (Day 2)

Name: KEY
Date:_____ Period:_____

1. A plane flies 62m east and 20m south from an airport. The pilot flies directly back to the airport. What bearing should he take? What bearing would a ship take? Round to the nearest tenth of a degree.



Airplane: $\tan\theta = \frac{20}{62}$

$\tan^{-1}\frac{20}{62} = 17.9°$

Bearing angle $= 180° + 90° + 17.9°$

$= 287.9°$ → 287.9°

Ship: N ? W

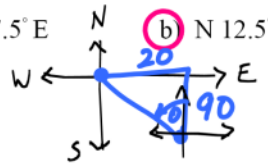
$\tan\theta = \frac{62}{20}$

$\tan^{-1}\frac{62}{20} = 72.1°$

N 72.1 °W

2. A ship is 90 miles south and 20 miles east of port. If the captain wants to travel directly to port, what bearing should be taken?

a) S 77.5° E b) N 12.5° W c) N 77.5° E d) S 12.5° W e) None of these



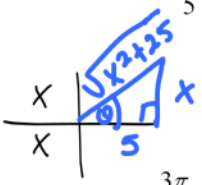
$\tan\theta = \frac{20}{90}$ $\tan^{-1}\frac{20}{90}$ $\theta = 12.5°$

3. Evaluate: $\arctan(1)$ $\tan? = 1$

a) $\frac{\pi}{4}$ b) $-\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) $\frac{7\pi}{4}$ e) None of these

4. Evaluate: $\sin\left(\arctan\frac{x}{5}\right)$

a) $\frac{x}{x+5}$ b) $\frac{x}{\sqrt{x^2+25}}$ c) $\frac{5}{\sqrt{x^2+25}}$ d) $\frac{\sqrt{25-x^2}}{5}$ e) None of these



$c^2 = x^2 + 5^2$

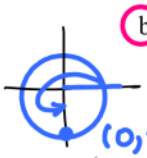
$c^2 = x^2 + 25$

$c = \sqrt{x^2+25}$

5. Evaluate: $\arcsin\left(\sin\frac{3\pi}{2}\right)$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$? NO * inverse property does not apply

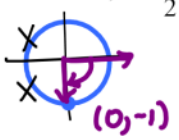
a) $\frac{\pi}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) $-\frac{3\pi}{2}$ e) None of these

$\sin\frac{3\pi}{2} = -1$



$\arcsin(-1) = -\frac{\pi}{2}$

$\sin? = -1$



6. Evaluate: $\tan\left(\arccos\left(-\frac{4}{5}\right)\right)$ $[-1, 1]$? ✓

a) $-\frac{4}{3}$ b) $\frac{4}{3}$ c) $-\frac{3}{4}$ d) $\frac{3}{4}$ e) None of these

$\tan\theta = -\frac{3}{4}$

7. Determine the period: $f(x) = -\frac{2}{3}\cos\left(\frac{x}{3} - \frac{1}{2}\right)$ $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$

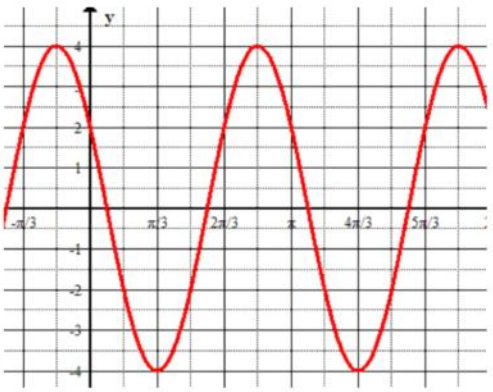
a) 6π b) $\frac{2\pi}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$ e) None of these

8. Which of the following is a vertical asymptote to the graph of $f(x)=\csc 3x$?

a) $x=\frac{\pi}{2}$ b) $x=\frac{3\pi}{2}$ c) $x=\frac{\pi}{3}$ d) $x=\frac{\pi}{4}$ e) None of these

* WORK on next pg.

9. Match the graph with the correct function.



a) $y=4\cos\left(3x-\frac{\pi}{2}\right)$ $3\left(x-\frac{\pi}{6}\right)$

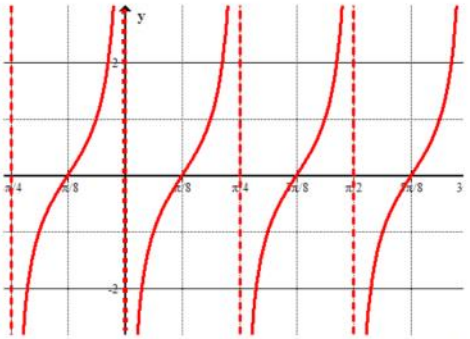
b) $y=4\cos\left(3x+\frac{\pi}{6}\right)$ $3\left(x+\frac{\pi}{18}\right)$

c) $y=4\sin\left(2x-\frac{\pi}{3}\right)$ $2\left(x-\frac{\pi}{6}\right)$

d) $y=4\cos\left(2x+\frac{\pi}{3}\right)$ $2\left(x+\frac{\pi}{6}\right)$

e) None of these

10. Match the correct function with the graph.



a) $y=\cot\left(x-\frac{\pi}{4}\right)$

b) $y=\tan\left(x-\frac{\pi}{4}\right)$ per $=\frac{\pi}{1}=\pi$

* c) $y=-\cot(4x)$

d) $y=\tan(4x)$ START: $4x-0=-\frac{\pi}{2}$ $4x=-\frac{\pi}{2}$ $x=-\frac{\pi}{8}$

* e) None of these

* START: $4x-0=0$ $4x=0$ ✓ $x=0$

* END: $4x-0=\pi$ $4x=\pi$ $x=\frac{\pi}{4}$ ✓

11. For given function $f(x)=-2\sec\left(\frac{\pi}{6}x+\frac{\pi}{3}\right)+5$ find:

$y=-2\cos\left(\frac{\pi}{6}x+\frac{\pi}{3}\right)+5$

$y=-2\cos\left[\frac{\pi}{6}(x+2)\right]+5$ ↖ FACTOR!

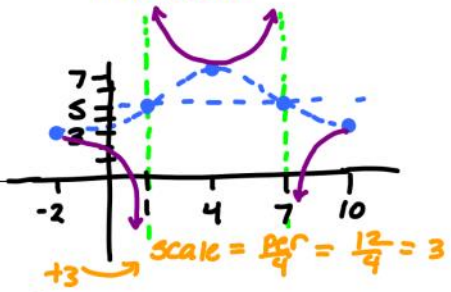
a) The phase shift: 2 units left

b) The range: $(-\infty,3]\cup[7,\infty)$

c) The period: $\frac{2\pi}{b}=\frac{2\pi}{\frac{\pi}{6}}=2\pi\cdot\frac{6}{\pi}=12$

d) The amplitude: none

e) All vertical asymptotes on $[-2,10]$: $x=1$ $x=7$



12. Sketch at least one full period. (show all work)

* see next pg.

a. $f(x)=4+\sin\left(2x-\frac{\pi}{6}\right)$

b. $g(x)=-2\cos\frac{\pi}{10}x+3$

c. $h(x)=-\tan(3x+\pi)$

d. $k(x)=\frac{3}{8}\cot\left(\frac{\pi}{4}x-\frac{\pi}{3}\right)$

e. $m(x)=3\sec(3x-\pi)$

f. $r(x)=3-\frac{3}{5}\csc\left(\frac{2\pi}{3}x+\frac{\pi}{4}\right)$

9. Which of the following is a vertical asymptote to the graph of $f(x)=\csc 3x$?

a) $x=\frac{\pi}{2}$ b) $x=\frac{3\pi}{2}$ c) $x=\frac{\pi}{3}$ d) $x=\frac{\pi}{4}$ e) None of these

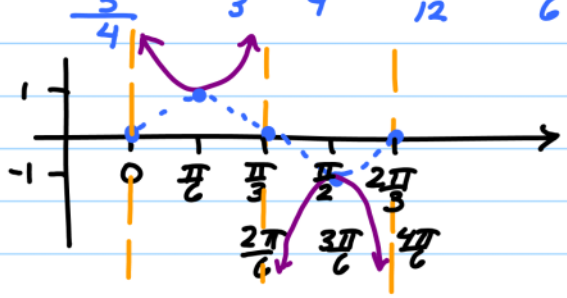
* graph $y = \sin 3x$

$\frac{2\pi}{b}$ period $= \frac{2\pi}{3}$

Start: $bx-c=0$ $3x=0$ $x=0$

END: $bx-c=2\pi$ $3x=2\pi$ $x=\frac{2\pi}{3}$

Period/4 Scale: $\frac{2\pi}{3} = \frac{2\pi}{3}\cdot\frac{1}{4} = \frac{2\pi}{12} = \frac{\pi}{6}$



12. Sketch at least one full period. (show all work)

a. $f(x)=4+\sin\left(2x-\frac{\pi}{6}\right)$

* up 4

$y = \sin\left(2\left(x-\frac{\pi}{12}\right)\right)+4$

* FACTOR

* Shift $\frac{\pi}{12}$ Right

period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ amplitude: $|a| = 1$

start: $bx-c=0$

$2x-\frac{\pi}{6}=0$

$\frac{1}{2}(2x)=\left(\frac{\pi}{6}\right)\frac{1}{2}$

$x=\frac{\pi}{12}$

end: $bx-c=2\pi$

$2x-\frac{\pi}{6}=2\pi$

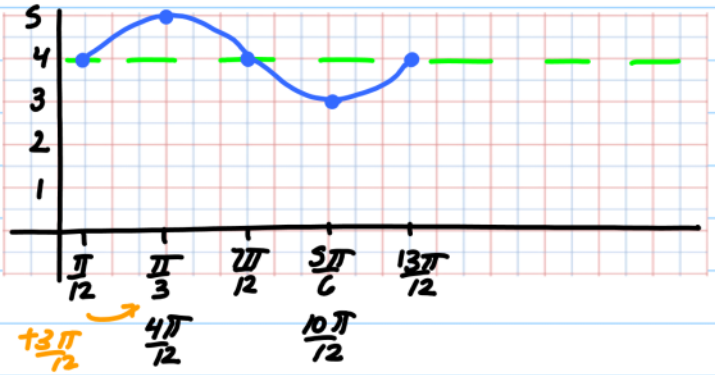
$2x=\frac{\pi}{6}+2\pi\frac{6}{6}$

$\frac{1}{2}(2x)=\left(\frac{13\pi}{6}\right)\frac{1}{2}$

$x=\frac{13\pi}{12}$

scale: $\frac{\text{per}}{4} = \frac{\pi}{4}\cdot\frac{3}{3}$

$=\frac{3\pi}{12}$



KEY points:

$(\frac{\pi}{12},4)$, $(\frac{\pi}{3},5)$,

$(\frac{7\pi}{12},4)$, $(\frac{5\pi}{6},3)$

$(\frac{13\pi}{12},4)$

$a=-2 \quad b=\frac{\pi}{10} \quad d=3$

b. $g(x)=-2\cos\frac{\pi}{10}x+3$ ← shift up 3

Reflect over x-axis

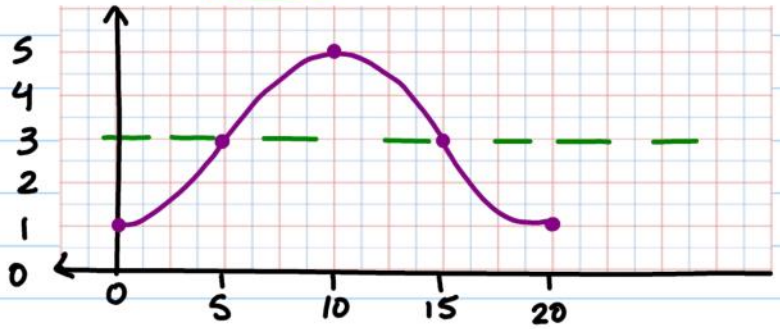
period $= \frac{2\pi}{b} \quad \frac{2\pi}{\frac{\pi}{10}} = 2\pi \cdot \frac{10}{\pi} = 20$ amplitude: $|a| = 2$

START: $bx-c=0$ END: $bx-c=2\pi$ scale: $\frac{per}{4}=\frac{20}{4}$

$\frac{10}{\pi}\left(\frac{\pi}{10}x\right)=(0)\frac{10}{\pi}$ $\frac{10}{\pi}\left(\frac{\pi}{10}x\right)=(2\pi)\frac{10}{\pi}$ $=5$

$x=0$ $x=20$

KEY POINTS:
(0,1), (5,3), (10,5)
(15,3), (20,1)



c. $h(x)=-\tan(3x+\pi)$

Reflect over x-axis

$y=-\tan\left(3\left(x+\frac{\pi}{3}\right)\right)$ * shift $\frac{\pi}{3}$ left

π factor!

amplitude: none

period: $\frac{\pi}{b}=\frac{\pi}{3}$

start: $bx-c=\frac{-\pi}{2}$ end: $bx-c=\frac{\pi}{2}$ scale: $\frac{per}{4}$

$3x+\pi=\frac{-\pi}{2}$ $3x+\pi=\frac{\pi}{2}$ $\frac{\frac{\pi}{3}}{4}=\frac{\pi}{3}\cdot\frac{1}{4}$

$3x=\frac{-\pi}{2}-\pi\cdot\frac{2}{2}$ $3x=\frac{\pi}{2}-\pi\cdot\frac{2}{2}$ $=\frac{\pi}{12}$

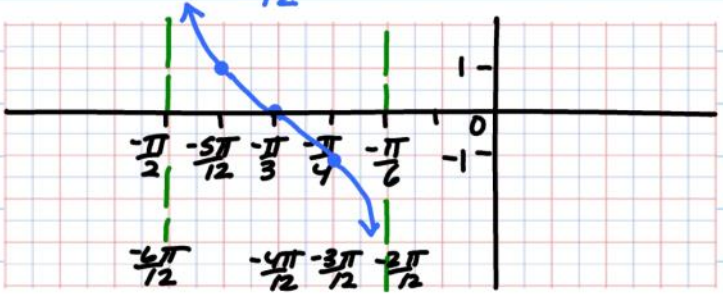
$\frac{1}{3}(3x)=\left(\frac{-3\pi}{2}\right)\frac{1}{3}$ $\frac{1}{3}(3x)=\left(\frac{-\pi}{2}\right)\frac{1}{3}$

$x=\frac{-\pi}{2}\cdot\frac{6}{6}$ $x=\frac{-\pi}{6}\cdot\frac{2}{2}$

$=\frac{-6\pi}{12}$ $=\frac{-2\pi}{12}$

KEY POINTS:
$\left(\frac{-5\pi}{12},1\right),\left(\frac{-\pi}{3},0\right),\left(\frac{-\pi}{4},-1\right)$

ASYMPTOTES:
$x=\frac{-\pi}{2}$ $x=\frac{-\pi}{6}$



d. $k(x)=\frac{3}{8}\cot\left(\frac{\pi}{4}x-\frac{\pi}{3}\right)$

$y=\frac{3}{8}\cot\left(\frac{\pi}{4}(x-\frac{4}{3})\right)$

* FACTOR ← shift $\frac{4}{3}$ right

amplitude: none Period: $\frac{\pi}{b}=\frac{\pi}{\frac{\pi}{4}}=\pi\cdot\frac{4}{\pi}=4$

Start: $bx-c=0$

$\frac{\pi}{4}x-\frac{\pi}{3}=0$

$\frac{4}{\pi}\left(\frac{\pi}{4}x\right)=\left(\frac{\pi}{3}\right)\frac{4}{\pi}$

$x=\frac{4}{3}$

END: $bx-c=\pi$

$\frac{\pi}{4}x-\frac{\pi}{3}=\pi$

$\frac{\pi}{4}x=\pi\cdot\frac{3}{3}+\frac{\pi}{3}$

$\frac{4}{\pi}\left(\frac{\pi}{4}x\right)=\left(\frac{4\pi}{3}\right)\frac{4}{\pi}$

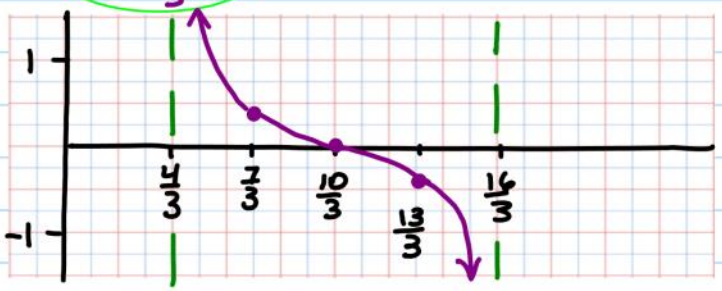
$x=\frac{16}{3}$

Scale: $\frac{per}{4}=\frac{4}{4}$

$=1$ or $\frac{3}{3}$

KEY points:
$(\frac{7}{3},\frac{3}{8})$, $(\frac{10}{3},0)$,
$(\frac{13}{3},-\frac{3}{8})$

ASYMPTOTES:
$x=\frac{4}{3}$ $x=\frac{16}{3}$



e. $m(x)=3\sec(3x-\pi)$

$y=3\cos(3x-\pi)$

* graph reciprocal 1st!

$y=3\cos(3(x-\frac{\pi}{3}))$

↑ FACTOR

* Shift $\frac{\pi}{3}$ Right

amp: none

per: $\frac{2\pi}{b}=\frac{2\pi}{3}$

Start: $bx-c=0$

$3x-\pi=0$

$3x=\pi$

$x=\frac{\pi}{3}\cdot\frac{2}{2}$

or $\frac{2\pi}{6}$

END: $bx-c=2\pi$

$3x-\pi=2\pi$

$\frac{3x}{3}=\frac{3\pi}{3}$

$x=\pi\cdot\frac{6}{6}$

or $\frac{6\pi}{6}$

Scale: $\frac{per}{4}$

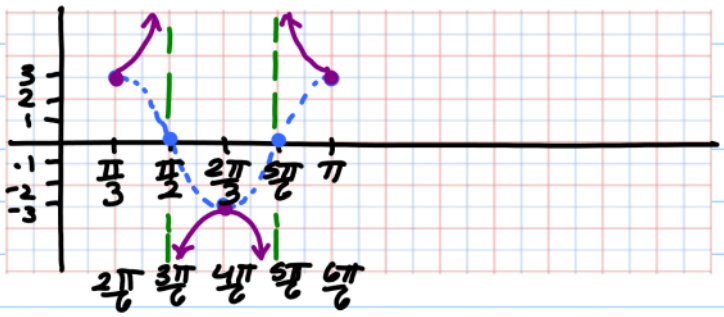
$\frac{\frac{2\pi}{3}}{4}=\frac{2\pi}{3}\cdot\frac{1}{4}$

$=\frac{\pi}{6}$

e. $m(x)=3\sec(3x-\pi)$ (continued)

ASYMPTOTES:
$x=\frac{\pi}{2}$ $x=\frac{5\pi}{6}$

POINTS:
$(\frac{\pi}{3},3),(\frac{2\pi}{3},-3),(\pi,3)$



* Challenge problem

f. $r(x)=3-\frac{3}{5}\csc\left(\frac{2\pi}{3}x+\frac{\pi}{4}\right)$

* graph reciprocal 1st!

$y=-\frac{3}{5}\sin\left(\frac{2\pi}{3}x+\frac{\pi}{4}\right)+3$ * shift 3up

$\frac{2\pi}{3}(x+\frac{3}{8})$ shift $\frac{3}{8}$ left

Reflect over x-axis

amp: none

period: $\frac{2\pi}{b}=\frac{2\pi}{\frac{2\pi}{3}}=2\pi\cdot\frac{3}{2\pi}=3$

START! $bx-c=0$

$\frac{2\pi}{3}x+\frac{\pi}{4}=0$

$\frac{3}{2\pi}\left(\frac{2\pi}{3}x\right)=\left(\frac{-\pi}{4}\right)\frac{3}{2\pi}$

$x=-\frac{3}{8}$

END: $bx-c=2\pi$

$\frac{2\pi}{3}x+\frac{\pi}{4}=2\pi$

$\frac{2\pi}{3}x=2\pi\cdot\frac{4}{4}-\frac{\pi}{4}$

$\frac{3}{2\pi}\left(\frac{2\pi}{3}x\right)=\left(\frac{7\pi}{4}\right)\frac{3}{2\pi}$

$x=\frac{21}{8}$

scale: per $\frac{1}{4}$ $=\frac{3}{4}$ or $\frac{6}{8}$

ASYMPTOTES:
$x=-\frac{3}{8}$, $x=\frac{9}{8}$, $x=\frac{21}{8}$

KEY points!
$(\frac{3}{8},2\frac{2}{5})$, $(\frac{15}{8},3\frac{3}{5})$