Tuesday, April 10, 2018
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Precalculus Name KEY
Quarter 3 Test Review #1

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER!

Solve the following problems. Unless otherwise specified, round **final** answers to the nearest **tenth**.

Section 5.1

1. Use the fundamental identities to simplify the expression $\tan^2 x - \tan^2 x \sin^2 x$

2. Use trigonometric identities to simplify $\dfrac{\cos^4 x - \sin^4 x}{\cos^2 x - \sin^2 x}$.

3. Use trigonometric identities to simplify $\cos t\left(1+\tan^2 t\right)$.

Section 5.2

4. Verify the identity: $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = 1$

5. Verify the identity: $\dfrac{1}{1-\sin\alpha} + \dfrac{1}{1+\sin\alpha} = 2\sec^2\alpha$

Section 5.3

Determine the number of solutions. Justify your answer.

6. a) $2\sin x + \sqrt{3} = 0$ b) $2\sin 3x + \sqrt{3} = 0$

Find all **exact** solutions of the following equations in the interval $[0, 2\pi)$:

7. a) $2\sin x + \sqrt{3} = 0$ b) $2\sin 3x + \sqrt{3} = 0$

8. $(3\tan^2 x + 1)(\tan^2 x - 3) = 0$

9. $\cos^3 x = \cos x$

Section 5.4

Use the appropriate sum or difference formula to find the **exact** values of the expressions in #10 – 11:

10. $\sin 255^\circ$, $\cos 255^\circ$, $\tan 255^\circ$

11. $\sin\frac{\pi}{12}$, $\cos\frac{\pi}{12}$, $\tan\frac{\pi}{12}$

12. Find the **exact** value for $\dfrac{\tan 325^\circ - \tan 25^\circ}{1+\tan 325^\circ \tan 25^\circ}$.

13. Simplify the expression $\cos 146^\circ \cos 11^\circ + \sin 146^\circ \sin 11^\circ$, and **evaluate**, if possible.

14. Given $\cot u = \frac{2}{5}$, $0 < u < \frac{\pi}{2}$, and $\cos v = -\frac{3}{5}$, $\pi < v < \frac{3\pi}{2}$, find $\tan(u+v)$

Section 5.5

15. Given $\tan\theta = \frac{3}{4}$ and $\sin\theta < 0$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

16. Find the **exact** solutions in the interval $[0, 2\pi)$ of $\sin 2x + \sin x = 0$.

17. Use a half-angle formula to find the **exact** value of $\cos 157^\circ 30'$.

18. Given $\tan u = -\frac{4}{3}$, and $\sin u < 0$, find $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, and $\tan\frac{u}{2}$.

19. Simplify by expressing each as a function of a **single** angle:

i) $\dfrac{2\tan 47^o}{1-\tan^2 47^o}$ ii) $-\sqrt{\dfrac{1-\cos 10x}{2}}$ iii) $\dfrac{1-\cos 18^o}{\sin 18^o}$

Precalculus
Quarter 3 Test Review #1

Section 5.1

1. Use the fundamental identities to simplify the expression $\tan^2 x - \tan^2 x \sin^2 x$

$= \tan^2 x(1-\sin^2 x)$
$= \tan^2 x(\cos^2 x)$
$= \dfrac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} = \boxed{\sin^2 x}$

2. Use trigonometric identities to simplify $\dfrac{\cos^4 x - \sin^4 x}{\cos^2 x - \sin^2 x}$.

$\dfrac{(\cos^2 x+\sin^2 x)\cancel{(\cos^2 x-\sin^2 x)}}{\cancel{(\cos^2 x-\sin^2 x)}} = \cos^2 x + \sin^2 x = \boxed{1}$

3. Use trigonometric identities to simplify $\cos t\left(1+\tan^2 t\right)$.

$= \cos t \sec^2 t = \dfrac{1}{\sec t} \cdot \sec^2 t = \boxed{\sec t}$

Section 5.2

4. Verify the identity: $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = \underline{1}$

$\dfrac{\frac{1}{\cos}}{\cos} - \dfrac{\tan\theta}{\frac{1}{\tan\theta}} = 1$

$\dfrac{1}{\cos\theta} \cdot \dfrac{1}{\cos\theta} - \tan\theta \cdot \tan\theta =$

$\dfrac{1}{\cos^2\theta} - \tan^2\theta =$

$\sec^2\theta - \tan^2\theta =$

$1 = 1$

5. Verify the identity:

$\dfrac{(1+\sin\alpha)}{(1+\sin\alpha)}\dfrac{1}{1-\sin\alpha} + \dfrac{1}{1+\sin\alpha}\dfrac{(1-\sin\alpha)}{(1-\sin\alpha)} = 2\sec^2\alpha$

$\dfrac{1+\sin\alpha + 1-\sin\alpha}{(1+\sin\alpha)(1-\sin\alpha)} = 2\sec^2\alpha$

$\dfrac{2}{1-\sin^2\alpha} =$

$\dfrac{2}{\cos^2\alpha} =$

$2\sec^2\alpha = 2\sec^2\alpha$

Section 5.3

Determine the number of solutions. Justify your answer.

6. a) $2\sin x + \sqrt{3} = 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

S | A
T | C

$\boxed{\text{2 solutions since sine is neg in QIII + QIV}}$

b) $2\sin 3x + \sqrt{3} = 0$

$2\sin 3x = -\sqrt{3}$

$\sin 3x = -\frac{\sqrt{3}}{2}$

Horizontal shrink by $\frac{1}{3}$

so 3 times as many solutions

$\boxed{3(2) = 6 \text{ solutions}}$

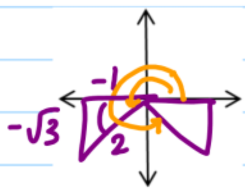
Find all **exact** solutions of the following equations in the interval $[0, 2\pi)$:

7. a) $2\sin x + \sqrt{3} = 0$

S | A
T | C

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$



$\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$

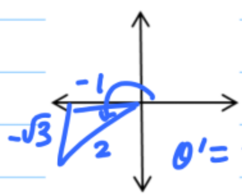
$\theta' = \frac{\pi}{3}$

b) $2\sin 3x + \sqrt{3} = 0$

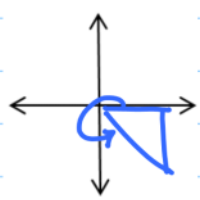
S | A
T | C

$2\sin 3x = -\sqrt{3}$

$\sin 3x = -\frac{\sqrt{3}}{2}$



$\theta' = \frac{\pi}{3}$



$\frac{3x}{3} = \frac{1}{3} \cdot \frac{4\pi}{3} + \frac{2\pi n}{3}$

$x = \frac{4\pi}{9} + \frac{2\pi}{3}n \cdot \frac{3}{3}$

$\boxed{\frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}}$

$\frac{3x}{3} = \frac{1}{3} \cdot \frac{5\pi}{3} + \frac{2\pi n}{3}$

$x = \frac{5\pi}{9} + \frac{2\pi n}{3} \cdot \frac{3}{3}$

$\boxed{\frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}}$

Find all **exact** solutions of the following equations in the interval $[0, 2\pi)$:

8. $(3\tan^2 x + 1)(\tan^2 x - 3) = 0$

$3\tan^2 x + 1 = 0$

$\tan^2 x = -\frac{1}{3}$

$\tan x = \pm\sqrt{-\frac{1}{3}}$

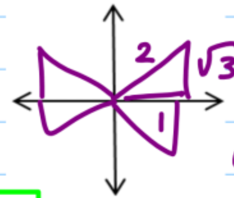
* not possible
* can't take $\sqrt{\ }$ of neg. #

$\tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$



$\theta' = \frac{\pi}{3}$

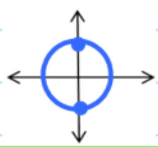
9. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x(\cos x + 1)(\cos x - 1) = 0$

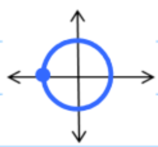
$\cos x = 0$



$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\cos x + 1 = 0$

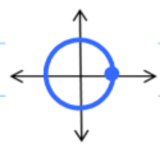
$\cos x = -1$



$\boxed{x = \pi}$

$\cos x - 1 = 0$

$\cos x = 1$

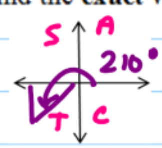


$\boxed{x = 0}$

Section 5.4

Use the appropriate sum or difference formula to find the **exact** values of the expressions in #10 – 11:

10. $\sin 255^\circ$, $\cos 255^\circ$, $\tan 255^\circ$



$\theta' = 30^\circ$

$\sin 255^\circ = \sin(210^\circ + 45^\circ) = \sin 210^\circ \cos 45^\circ + \cos 210^\circ \sin 45^\circ$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$

$\cos 255^\circ = \cos(210^\circ + 45^\circ) = \cos 210^\circ \cos 45^\circ - \sin 210^\circ \sin 45^\circ$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{6} + \sqrt{2}}{4}}$

$$\tan 255° = \tan(210° + 45°) = \frac{\tan 210° + \tan 45°}{1 - \tan 210° \tan 45°}$$

$$= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}(1)} = \frac{\left(\frac{\sqrt{3}}{3} + 1\right)3}{\left(1 - \frac{\sqrt{3}}{3}\right)3} = \frac{(\sqrt{3}+3)}{(3-\sqrt{3})} \cdot \frac{(3+\sqrt{3})}{(3+\sqrt{3})}$$

$$= \frac{3\sqrt{3} + 3 + 9 + 3\sqrt{3}}{9 - 3} = \frac{12 + 6\sqrt{3}}{6} = \boxed{2 + \sqrt{3}}$$

$\frac{\pi}{6} = \frac{2\pi}{12}$

$\frac{3}{3}\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{4}{4}\frac{\pi}{3} = \frac{4\pi}{12}$

11. $\sin\frac{\pi}{12}$, $\cos\frac{\pi}{12}$, $\tan\frac{\pi}{12}$

$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

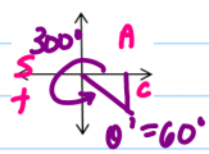
$$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}(1)}$$

$$= \frac{(\sqrt{3}-1)}{(1+\sqrt{3})} \cdot \frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3} = \frac{2\sqrt{3} - 4}{-2} = \boxed{-\sqrt{3} + 2}$$

12. Find the **exact** value for $\frac{\tan 325° - \tan 25°}{1 + \tan 325° \tan 25°}$.

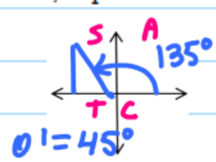
$$= \tan(325° - 25°) = \tan 300°$$

$$= \boxed{-\sqrt{3}}$$



13. Simplify the expression $\cos 146° \cos 11° + \sin 146° \sin 11°$, and **evaluate**, if possible.

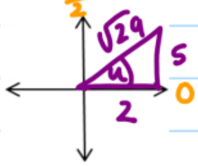
$$= \cos(146° - 11°) = \cos 135° = \boxed{-\frac{\sqrt{2}}{2}}$$



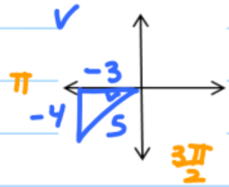
14. Given $\cot u = \frac{2}{5}$, $0 < u < \frac{\pi}{2}$, and $\cos v = -\frac{3}{5}$, $\pi < v < \frac{3\pi}{2}$, find $\tan(u+v)$

$\frac{9}{0}$

u



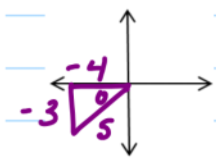
v



$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\frac{3}{3}\cdot\frac{5}{2} + \frac{-4}{-3}\cdot\frac{2}{2}}{1 - \left(\frac{5}{2}\right)\left(\frac{-4}{-3}\right)} = \frac{\frac{15}{6} + \frac{8}{6}}{1 - \frac{10}{3}}$$

$$= \frac{\frac{23}{6}}{-\frac{7}{3}} = \frac{23}{6}\cdot\frac{3}{-7} = \boxed{-\frac{23}{14}}$$

Section 5.5



15. Given $\tan\theta = \frac{3}{4}$ and $\sin\theta < 0$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

pos neg

S | A
T | C

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \boxed{\frac{24}{25}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2}\cdot\frac{16}{7}$$

$$= \boxed{\frac{24}{7}}$$

16. Find the **exact** solutions in the interval $[0, 2\pi)$ of $\sin 2x + \sin x = 0$.

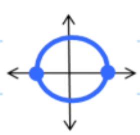
* Double-Angle formula

$2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ $\quad$ $2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$



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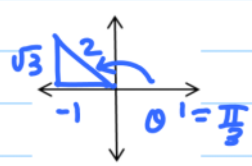
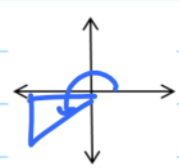
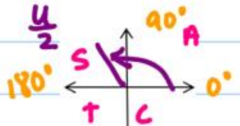




$\boxed{x = 0, \pi}$ $\quad$ $\boxed{x = \frac{2\pi}{3}}$ $\quad$ $\boxed{x = \frac{4\pi}{3}}$

17. Use a half-angle formula to find the **exact** value of $\cos 157°30'$.

$$\cos 157°30' = \cos\left(\frac{315°}{2}\right) = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\cos 315°}{2}}$$



$\theta' = 45°$

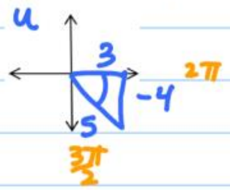
* $u/2$ is in Q II
* cos is neg in QII

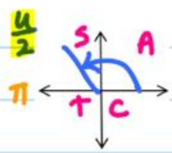
$$= -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$$

or $\boxed{-\frac{1}{2}\sqrt{2+\sqrt{2}}}$

18. Given $\tan u = -\frac{4}{3}$, and $\sin u < 0$, find $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, and $\tan\frac{u}{2}$.





$$\frac{1}{2}\cdot\frac{3\pi}{2} < \frac{u}{2} < \frac{2\pi}{2}$$

$$\frac{3\pi}{4} < \frac{u}{2} < \pi$$

* $\frac{u}{2}$ is in Q II
* sin pos
* cos neg

$$\sin\frac{u}{2} = +\sqrt{\frac{1-\cos u}{2}} = +\sqrt{\frac{1-\frac{3}{5}}{2}} = +\sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{2}{5}\cdot\frac{1}{2}}$$

$$= \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$$

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{3}{5}}{2}} = -\sqrt{\frac{\frac{8}{5}}{2}} = -\sqrt{\frac{\cancel{8}^{4}}{5}\cdot\frac{1}{\cancel{2}_{1}}}$$

$$= -\frac{\sqrt{4}}{\sqrt{5}} = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1-\frac{3}{5}}{-\frac{4}{5}} = \frac{\frac{2}{5}}{-\frac{4}{5}} = \frac{2}{\cancel{5}}\cdot\left(-\frac{\cancel{5}}{\cancel{4}_{2}}\right) = \boxed{-\frac{1}{2}}$$

19. Simplify by expressing each as a function of a **single** angle:

i) $\dfrac{2\tan 47^{o}}{1-\tan^{2}47^{o}}$

$= \tan 2\theta$

$= \tan 2(47^\circ)$

$= \boxed{\tan 94^\circ}$

ii) $-\sqrt{\dfrac{1-\cos 10x}{2}}$

$= \left|\sin \frac{\theta}{2}\right|$

$= -\left|\sin \frac{10x}{2}\right|$

$= \boxed{-\left|\sin 5x\right|}$

iii) $\dfrac{1-\cos 18^{o}}{\sin 18^{o}}$

$= \tan \frac{\theta}{2}$

$= \tan \frac{18^\circ}{2}$

$= \boxed{\tan 9^\circ}$