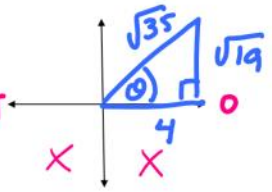
Tuesday, January 30, 2018
8:10 AM

Precalculus Midterm 2018 Exam Review #2 Questions Name KEY

Complete the following problems on a separate sheet of paper

1. Evaluate $\sin\left(\arccos\frac{4}{\sqrt{35}}\right)$.



$[-1,1]$? ✓

$c^2 = a^2 + b^2$

$(\sqrt{35})^2 = (4)^2 + b^2$

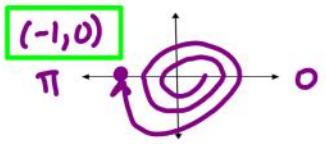
$35 = 16 + b^2$

$19 = b^2$

$b = \sqrt{19}$

$\sin\theta = \frac{\sqrt{19}}{\sqrt{35}} \cdot \frac{\sqrt{35}}{\sqrt{35}} = \frac{\sqrt{665}}{35}$

2. Find the point (x, y) on the unit circle which corresponds to the real number $t = -5\pi$.



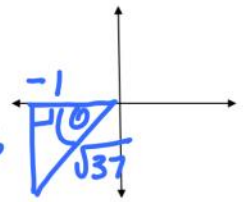
3. Find $\sec\theta$ given that θ lies in quadrant III and $\tan\theta = 6$.

$c^2 = a^2 + b^2$

$c^2 = (-1)^2 + (-6)^2$

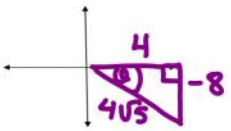
$c^2 = 37$

$c = \sqrt{37}$



$\sec\theta = \frac{\sqrt{37}}{-1} = -\sqrt{37}$

4. Find the six trig functions of the angle θ (in standard position) whose terminal side passes through $(4,-8)$.



$c^2 = (4)^2 + (-8)^2$

$c^2 = 80$

$c = \sqrt{80}$

$c = 4\sqrt{5}$

$\sin\theta = \frac{-8}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{4}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{-8}{4} = -2$

$\csc\theta = -\frac{\sqrt{5}}{2}$

$\sec\theta = \sqrt{5}$

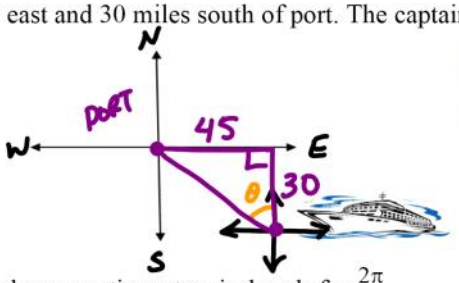
$\cot\theta = -\frac{1}{2}$

5. Find the amplitude and period of $y = -6\cos\left(\frac{x}{3} + \pi\right) - 2$.

amplitude $= |-6| = 6$

period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot \frac{3}{1} = 6\pi$

6. A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?



N ? W

$\tan\theta = \frac{45}{30}$

$\tan^{-1}\frac{45}{30}$

$\theta \approx 56.31°$

N 56.31° W

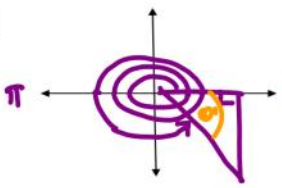
7. Find one positive and one negative coterminal angle for $\frac{2\pi}{9}$

* Add or subtract multiples of 2π

$\frac{2\pi}{9} + 2\pi \cdot \frac{9}{9} = \frac{2\pi}{9} + \frac{18\pi}{9} = \frac{20\pi}{9}$

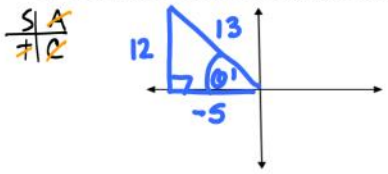
$\frac{2\pi}{9} - 2\pi \cdot \frac{9}{9} = \frac{2\pi}{9} - \frac{18\pi}{9} = -\frac{16\pi}{9}$

8. Find the reference angle for $\frac{17\pi}{3}$



$\theta' = \frac{\pi}{3}$

9. Find the 5 remaining trigonometric functions (ratios) given $\tan\theta = -\frac{12}{5}$ and $\sin\theta > 0$.

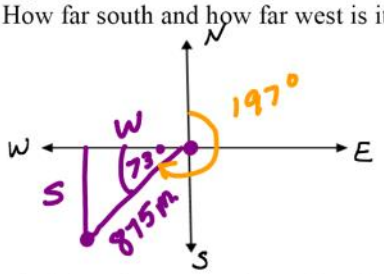


$\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{-5}{13}$ $\quad \sec\theta = -\frac{13}{5}$

$\tan\theta = \frac{-12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

10. A plane flies at a bearing of 197° W after leaving an airport at noon at a speed of 350 miles per hour. How far south and how far west is it from its point of departure at 2:30 PM ?



350 mph · 2.5 hrs = 875 miles

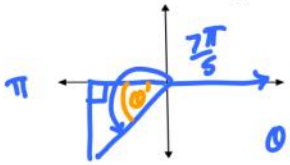
270° − 197° = 73°

$\sin 73° = \frac{S}{875}$ $\quad \cos 73° = \frac{W}{875}$

$875 \sin 73° = S$ $\quad 875 \cos 73° = W$

$S = 836.77$ miles south $\quad W = 255.83$ miles west

11. Find the reference angle θ' for $\theta = \frac{7\pi}{5}$



$\theta' = \frac{2\pi}{5}$

$\theta' = \frac{7\pi}{5} - \frac{5\pi}{5} = \frac{2\pi}{5}$

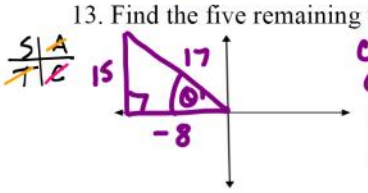
12. Evaluate $\csc 3.92$. Round to 4 decimal places.

* Radians mode! (no °)

$\frac{1}{\sin 3.92}$

$= -1.4242$

13. Find the five remaining trig functions given $\cot\theta = -\frac{8}{15}$ and $\cos\theta < 0$.



$c^2 = a^2 + b^2$

$c^2 = (15)^2 + (-8)^2$

$c^2 = 289$

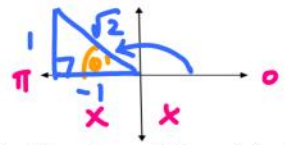
$c = 17$

$\sin\theta = \frac{15}{17}$ $\quad \csc\theta = \frac{17}{15}$

$\cos\theta = \frac{-8}{17}$ $\quad \sec\theta = -\frac{17}{8}$

$\tan\theta = -\frac{15}{8}$ $\quad \cot\theta = -\frac{8}{15}$

14. Find the exact value of $\arccos\left(-\frac{\sqrt{2}}{2}\right)$.



$\frac{\sqrt{2}}{2}$

$\frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$

[-1,1]? ✓

$\theta' = \frac{\pi}{4}$ $\quad \theta = \frac{3\pi}{4}$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

15. Sketch one full period of $y = \tan\left(2x - \frac{\pi}{4}\right)$.

shift $\frac{\pi}{8}$ Rt

$2\left(x - \frac{\pi}{8}\right)$

Period $= \frac{\pi}{b} = \frac{\pi}{2}$

Start: $bx - c = -\frac{\pi}{2}$

$2x - \frac{\pi}{4} = -\frac{\pi}{2}$

$2x = -\frac{\pi}{2} \cdot \frac{2}{2} + \frac{\pi}{4}$

$2x = -\frac{2\pi}{4} + \frac{\pi}{4}$

$\frac{1}{2}2x = -\frac{\pi}{4} \cdot \frac{1}{2}$

$x = -\frac{\pi}{8}$

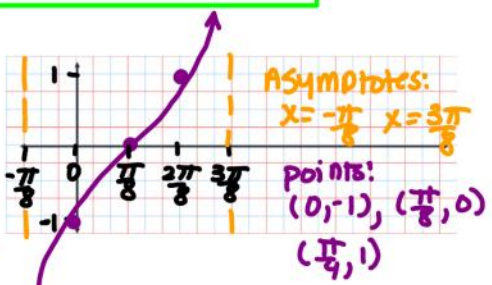
END: $bx - c = \frac{\pi}{2}$

$2x - \frac{\pi}{4} = \frac{\pi}{2}$

$2x = \frac{\pi}{2} \cdot \frac{2}{2} + \frac{\pi}{4}$

$\frac{1}{2}2x = \frac{3\pi}{4} \cdot \frac{1}{2}$

$x = \frac{3\pi}{8}$



16. Sketch one full period of $f(x) = -\csc(x + \pi) + 1$.

← shift up 1

← shift π left

Reflect over x-axis

* graph $g(x) = -\sin(x + \pi) + 1$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Start: $bx - c = 0$

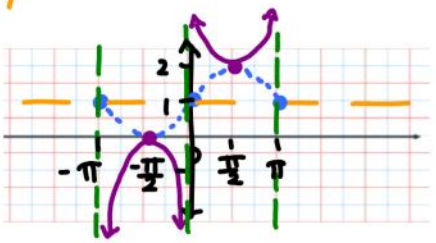
$x + \pi = 0$ $\quad x = -\pi$

END: $bx - c = 2\pi$

$x + \pi = 2\pi$ $\quad x = \pi$

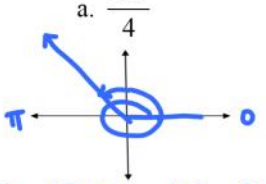
Asymptotes: $x = -\pi$, $x = 0$, $x = \pi$

Points: $(-\frac{\pi}{2}, 0)$, $(\frac{\pi}{2}, 2)$



17. For the following, sketch the angle in standard position and determine one positive and one negative coterminal angle

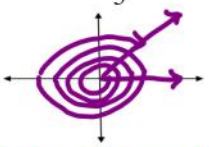
a. $\frac{11\pi}{4}$



$\frac{11\pi}{4} - 2\pi\frac{4}{4} = \frac{11\pi}{4} - \frac{8\pi}{4} = \boxed{\frac{3\pi}{4}}$

$\frac{3\pi}{4} - 2\pi\frac{4}{4} = \frac{3\pi}{4} - \frac{8\pi}{4} = \boxed{\frac{-5\pi}{4}}$

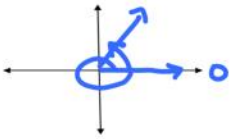
b. $-\frac{23\pi}{3}$



$-\frac{23\pi}{3} + 2\pi\frac{3}{3} = -\frac{23\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{-17\pi}{3}}$

$-\frac{17\pi}{3} + \frac{6\pi}{3} = -\frac{11\pi}{3} + \frac{6\pi}{3} = -\frac{5\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{\pi}{3}}$

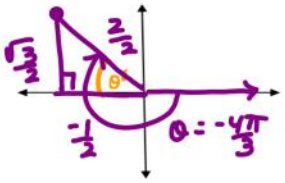
c. 405°



$405^\circ - 360^\circ = \boxed{45^\circ}$

$45^\circ - 360^\circ = \boxed{-315^\circ}$

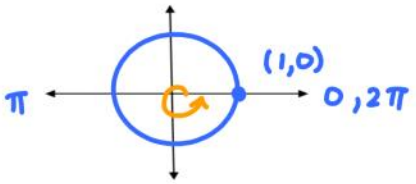
18. Find the point (x, y) on the unit circle which corresponds to the real number $t = -\frac{4\pi}{3}$.



$\theta' = \frac{\pi}{3}$

$\boxed{\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$

19. Evaluate, if possible, the 6 trigonometric functions of the real number $t = 2\pi$

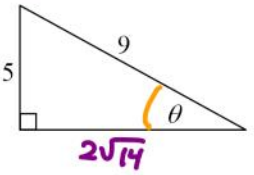


y: $\sin\theta = 0$ $\quad \csc\theta = \frac{1}{0} = \text{undef.}$

x: $\cos\theta = 1$ $\quad \sec\theta = 1$

x·y: $\tan\theta = \frac{0}{1} = 0$ $\quad \cot\theta = \frac{1}{0} = \text{undef.}$

20. Find the exact values of all 6 trigonometric functions of the angle in the following right triangle:



$c^2 = a^2 + b^2$

$9^2 = 5^2 + b^2$

$b^2 = 56$

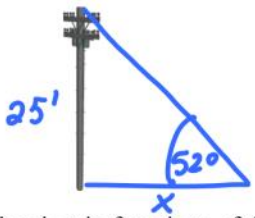
$b = 2\sqrt{14}$

$\sin\theta = \boxed{\frac{5}{9}}$ $\quad \csc\theta = \boxed{\frac{9}{5}}$

$\cos\theta = \boxed{\frac{2\sqrt{14}}{9}}$ $\quad \sec\theta = \frac{9}{2\sqrt{14}}\cdot\frac{\sqrt{14}}{\sqrt{14}} = \boxed{\frac{9\sqrt{14}}{28}}$

$\tan\theta = \frac{5}{2\sqrt{14}}\cdot\frac{\sqrt{14}}{\sqrt{14}} = \boxed{\frac{5\sqrt{14}}{28}}$ $\quad \cot\theta = \boxed{\frac{2\sqrt{14}}{5}}$

21. A wire runs from the ground to the top of a 25-foot telephone pole. The angle formed between the wire and the ground is 52^0. How far from the base of the pole is the wire attached to the ground?



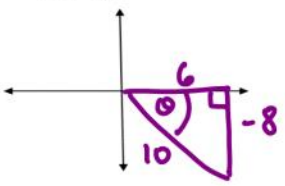
$\tan 52^\circ = \frac{25}{x}$

$x\tan 52^\circ = 25$

$x = \frac{25}{\tan 52^\circ}$

$\boxed{x = 19.53 \text{ feet}}$

22. Find the six trig functions of the angle θ (in standard position) whose terminal side passes through $(6, -8)$



$c^2 = 6^2 + (-8)^2$

$c^2 = 100$

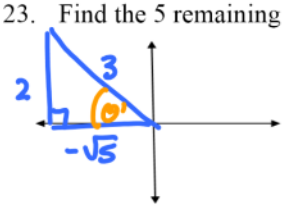
$c = 10$

$\sin\theta = \frac{-8}{10} = \boxed{\frac{-4}{5}}$ $\quad \csc\theta = \boxed{-\frac{5}{4}}$

$\cos\theta = \frac{6}{10} = \boxed{\frac{3}{5}}$ $\quad \sec\theta = \boxed{\frac{5}{3}}$

$\tan\theta = \frac{-8}{6} = \boxed{\frac{-4}{3}}$ $\quad \cot\theta = \boxed{-\frac{3}{4}}$

23. Find the 5 remaining trigonometric functions (ratios) given $\csc\theta = \frac{3}{2}$ and $\cos\theta < 0$.



$c^2 = a^2 + b^2$

$(3)^2 = (2)^2 + b^2$

$5 = b^2$

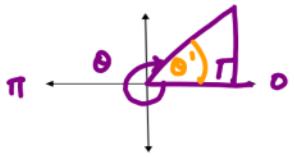
$b = \sqrt{5}$

$\sin\theta = \frac{2}{3}$ $\quad \csc\theta = \frac{3}{2}$

$\cos\theta = \frac{-\sqrt{5}}{3}$ $\quad \sec\theta = \frac{-3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$

$\tan\theta = \frac{2}{-\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$ $\quad \cot\theta = \frac{-\sqrt{5}}{2}$

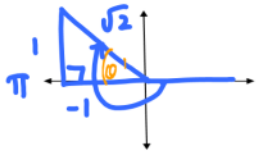
24. Sketch in standard position and find the reference angle for $-\frac{8\pi}{5}$.



$\theta' = \frac{2\pi}{5}$

* Reference Angle = Distance closest to x-axis
* Always positive
* Always acute

25. Evaluate sine, cosine, and tangent of $-\frac{5\pi}{4}$ without a calculator.



$\theta' = \frac{\pi}{4}$

$\sin -\frac{5\pi}{4} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos -\frac{5\pi}{4} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\tan -\frac{5\pi}{4} = \frac{1}{-1} = -1$

For # 26 & 27, find amplitude, period, range, phase and vertical shifts where applicable. Then, sketch one full period for each function. Identify all key elements.

26. $y = 8\cos\left(\frac{x}{4}\right) + 1$.

Range: [-7, 9]

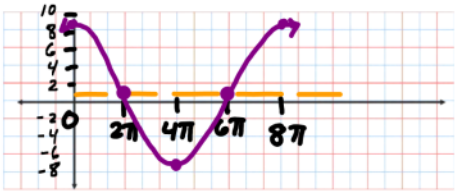
Amplitude = 8

Period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$

Vertical shift: up 1 $\quad$ Phase shift: none

START: $bx - c = 0$ $\quad \frac{x}{4} = 0$ $\quad x = 0$

END: $bx - c = 2\pi$ $\quad \frac{x}{4} = 2\pi$ $\quad x = 8\pi$



Points: (0, 9), (2π, 1), (4π, -7), (8π, 9)

27. $y = 2\cot(2t)$.

amplitude: none

period = $\frac{\pi}{b} = \frac{\pi}{2}$

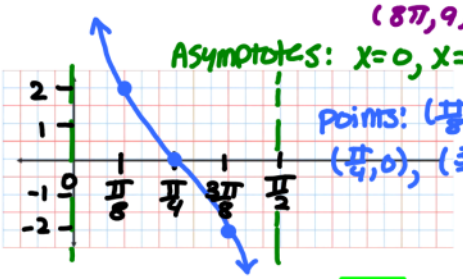
range: $(-\infty, \infty)$

phase shift: none

vert. shift: none

START: $bx - c = 0$ $\quad 2t = 0$ $\quad t = 0$

END: $bx - c = \pi$ $\quad 2t = \pi$ $\quad t = \frac{\pi}{2}$

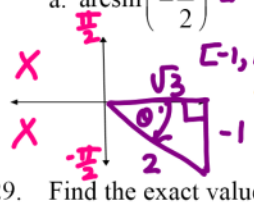


Asymptotes: $x = 0$, $x = \frac{\pi}{2}$

Points: $(\frac{\pi}{8}, 2)$, $(\frac{\pi}{4}, 0)$, $(\frac{3\pi}{8}, -2)$

28. Evaluate without a calculator:

a. $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

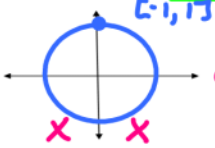


[-1, 1] ✓

* At what angle is $\frac{\text{opp}}{\text{hyp}} = -\frac{1}{2}$?

b. $\cos^{-1}(0) = \frac{\pi}{2}$

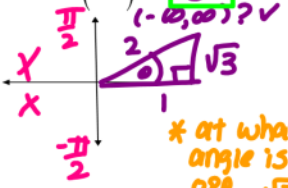
[-1, 1] ?✓



At what angle is $\cos\theta = 0$?

c. $\arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$

$(-\infty, \infty)$?✓

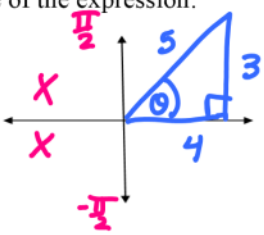


* at what angle is $\frac{\text{opp}}{\text{adj}} = \sqrt{3}$?

29. Find the exact value of the expression:

a. $\cos\left(\arctan\frac{3}{4}\right)$

$(-\infty, \infty)$?✓



$\cos\theta = \frac{4}{5}$

b. $\cot\left(\arcsin\left(-\frac{12}{13}\right)\right)$

[-1, 1] ✓

$\cot\theta = \frac{-5}{12}$