Friday, May 24, 2019
1:14 PM

KEY

Precalc — Ch. 3 Day 2: Properties of Natural Logs

Obj:

- To convert exponential & natural logarithmic functions
- To apply properties of logarithms to expand and condense expressions

Hwk: Ch 3. Day 1 HW WS

Major Assessment Thursday/Friday June 6/7

Do Now: Read the example in the box, then complete #1 & 2 below

Natural Logs: If $y=e^x$, then $x=\log_e y \rightarrow x=\ln y$

$\ln 9 = x \rightarrow \log_e 9 = x$ y=9 base exponent $\boxed{e^x=9}$

1) Rewrite in exponential form:

a. $\log_e 12 = x$ — $e^x = 12$

b. $\ln 12 = x$ — e — $e^x = 12$

c. $\ln 2.781 \approx 1$ — e — $e^1 \approx 2.781$

2) Rewrite in natural log form:

a. $e^8 = x$ — $\log_e x = 8$ — $\ln x = 8$

b. $e^1 \approx 2.781$ — $\log_e 2.781 \approx 1$ — $\ln 2.781 \approx 1$

c. $e^x = 1$ — $\log_e 1 = x$ — $\ln 1 = x$

* can you evaluate this? $x = 0$!

Class Notes:

Properties of Natural Logarithms:

1. $\ln 1 = 0$ because $e^0 = 1$
2. $\ln e = 1$ because $e^1 = e$
3. $\ln e^x = x$ and $e^{\ln x} = x$ (Inverse Prop.)
4. If $\ln x = \ln y$ then $x = y$ (One-to-One Prop.)

*Bases match
* so x must = y

These properties also work for base 10 (log)

Ex. 1) Apply the properties of logs to simplify:

a. $\ln_e e^{1/3} = \boxed{1/3}$

match

$e^x = e^{1/3}$

b. $e^{\ln_e 8} = \boxed{8}$

match

c. $15 \ln_e 1 = x$

$e^x = 1$

$x = 0$

$15(0) = \boxed{0}$

d. $3 - \ln_e e = x$

$e^x = e$

$x = 1$

$3 - 1 = \boxed{2}$

e. $5 + \ln_e 1 = x$

$e^x = 1$

$x = 0$

$5 + 0 = \boxed{5}$

f. $e^{\ln_e \pi}$

match

$= \boxed{\pi}$

g. $\ln_e e^1 + \ln_e e^7 - \ln_e e^{5x}$

$= 1 + 7 - 5x$

$= \boxed{8 - 5x}$

h. $e^0 + e^{\ln_e ☺}$

match

$\boxed{1 + ☺}$

Class Notes:

Ex.2) DESCRIBE how to simplify each:

a. $2^5 \cdot 2^3$

b. $\dfrac{2^5}{2^3}$

c. $\left(2^5\right)^3$

* bases match
* Add exponents

* subtract exponents

* multiply exponents

Apply your "rules" to simplify:

d. $(-3a^1b^5)(4a^1b^{-3})$

$= \boxed{-12a^2b^2}$

e. $\left(\dfrac{5x^3}{x^1}\right)^2 = (5x^2)^2$

$= \boxed{25x^4}$

f. $(2xy^5)^3$

$= 2^3x^3y^{15}$

$= \boxed{8x^3y^{15}}$

Reminder – logarithms are EXPONENTS, so the rules for exponents apply to logarithms – just in different forms:

☆Note: We will focus on natural Logarithms today☆

Properties of Logarithms [and Natural Logarithms]:

1. Product: $\log_a(uv) = \log_a u + \log_a v$ — $\ln(uv) = \ln u + \ln v$ *
2. Quotient: $\log_a \frac{u}{v} = \log_a u - \log_a v$ — $\ln \frac{u}{v} = \ln u - \ln v$
3. Power: $\log_a u^n = n \log_a u$ — $\ln u^n = n \ln u$

where a pos., a ≠ 1, n a real number, u & v pos. real #s

Ex. 3) Use the properties of logs to *rewrite and simplify* each expression.

a. $\ln\frac{100}{e} = \ln 100 - \ln e$

$= \boxed{\ln 100 - 1}$

e match

b. $\ln\frac{6}{e^2} = \ln 6 - \ln e^2$

$= \ln 6 - 2\ln e$

$= \boxed{\ln 6 - 2}$

e match

c. $\ln\left(5e^3\right)$

$= \ln 5 + \ln e^3$

$= \boxed{\ln 5 + 3}$

or $\ln 5 + \ln e^3$

$\ln 5 + 3\ln e$

$\ln 5 + 3(1)$

d. $\ln\left(3e^4\right)$

$= \ln 3 + \ln e^4$

$= \ln 3 + 4\ln e$

$= \boxed{\ln 3 + 4}$

Check! Is everything simplified as much as possible?

Rewriting Logarithmic Expressions – Condensing and Expanding

Ex. 4) Use the properties of logs to *expand* the expression as a sum, difference, and/or constant multiple of logs

a. $\ln 3x^2y$

$= \ln 3 + \ln x^2 + \ln y$

$= \boxed{\ln 3 + 2\ln x + \ln y}$

b. $\ln\frac{8\sqrt{x}}{e+1}$

$= \ln 8 + \ln x^{1/2} - \ln e - \ln 1$

$= \boxed{\ln 8 + \frac{1}{2}\ln x - 1} - 0$

c. $\log_e\left(\frac{1}{e^3}\right)$

$= \ln 1 - \ln e^3$

$= 0 - 3\ln e = 3(1) = \boxed{3}$

d. $\ln\sqrt{x^2(x+2)} = \ln\left(x^2(x+2)\right)^{1/2}$

$\frac{1}{2}\ln x^2 + \frac{1}{2}\ln(x+2)$

$\boxed{\ln x + \frac{1}{2}\ln(x+2)}$

Ex. 5) *Condense* the expression to the log of a single quantity:

a. $4\ln(x-4) - 2\ln x$

$= \boxed{\ln \frac{(x-4)^4}{x^2}}$

b. $\frac{1}{3}\ln x + 5\ln(x-3)$

$= \ln (x^{1/3})(x-3)^5$

$= \boxed{\ln\left(\sqrt[3]{x}\,(x-3)^5\right)}$

c. $\frac{1}{5}[\log_e x + \log_e(x-2)]$

$= \ln\left(x^{1/5}(x-2)^{1/5}\right)$

$\boxed{\begin{aligned} &= \ln \sqrt[5]{x(x-2)} \\ &= \ln \sqrt[5]{x^2-2x} \end{aligned}}$ OR

d. $2\ln 8 - 5\ln(m-4)$

$\ln \frac{8^2}{(m-4)^5}$

$= \boxed{\ln \frac{64}{(m-4)^5}}$

Closure:

Condense

$3\log_e x + 4\log_e y - 4\log_e z$

$\boxed{\ln \frac{x^3 y^4}{z^4}}$

Expand:

$\ln \frac{\sqrt{4x+1}}{6}$

$\ln(4x+1)^{1/2} - \ln 6$

$\boxed{\frac{1}{2}\ln(4x+1) - \ln 6}$

If time:

Find the exact value of each expression WITHOUT a calculator

a. $\log_e \sqrt[5]{e}$

$= \ln_e e^{1/5}$

$= \boxed{\frac{1}{5}}$

b. $\ln e^{12} + \ln e^{15}$

$= \ln e^{12} e^{15}$

$= \ln_e e^{27}$

$= \boxed{27}$

KEY

Homework: Ch. 3 Day 2: Properties of Natural Logs

In Exercises 23–28, use the properties of logarithms to rewrite and simplify the logarithmic expression.

27. $\ln(5e^6)$

$\ln 5 + \ln_e e^6 = \boxed{\ln 5 + 6}$

28. $\ln \frac{6}{e^2} = \ln 6 - \ln e^2$

$= \ln 6 - 2\ln_e e = \boxed{\ln 6 - 2}$

In Exercises 29–44, find the exact value of the logarithmic expression without using a calculator. (If this is not possible, state the reason.)

37. $\ln_e e^{4.5} = \boxed{4{,}5}$

38. $3 \ln_e e^4 = 3(4) = \boxed{12}$

39. $\ln \frac{1}{\sqrt{e}} = \ln 1 - \ln e^{1/2}$

$= 0 - \frac{1}{2} = \boxed{-\frac{1}{2}}$

40. $\ln \sqrt[4]{e^3} = \frac{1}{4}\ln e^3 = \frac{3}{4}\ln e = \frac{3}{4}(1)$

$= \boxed{\frac{3}{4}}$

41. $\ln_e e^2 + \ln_e e^5$

$= 2 + 5 = \boxed{7}$

42. $2 \ln_e e^6 - \ln_e e^5$

$= 2(6) - 5 = 12 - 5 = \boxed{7}$

In Exercises 45–66, use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

45. $\ln 4x$

$\boxed{\ln 4 + \ln x}$

51. $\ln \sqrt{z} = \ln z^{1/2}$

$\boxed{\frac{1}{2}\ln z}$

52. $\ln \sqrt[3]{t} = \ln t^{1/3}$

$= \boxed{\frac{1}{3}\ln t}$

53. $\ln xyz^2$

$\boxed{\ln x + \ln y + 2\ln z}$

58. $\ln \frac{6}{\sqrt{x^2+1}}$

$\boxed{\ln 6 - \frac{1}{2}\ln(x^2+1)}$

$\ln x^{1/3} - \ln y^{1/3}$

59. $\ln \sqrt[3]{\dfrac{x}{y}}$

$\boxed{\frac{1}{3}\ln x - \frac{1}{3}\ln y}$

60. $\ln \sqrt{\dfrac{x^2}{y^3}} = \ln\left(\dfrac{x^2}{y^3}\right)^{1/2} = \ln\left(\dfrac{x}{y^{3/2}}\right)$

$= \boxed{\ln x - \frac{3}{2}\ln y}$

In Exercises 67–84, condense the expression to the logarithm of a single quantity.

67. $\ln 2 + \ln x$

$\boxed{\ln(2x)}$

68. $\ln y + \ln t$

$\boxed{\ln(yt)}$

76. $2 \ln 8 + 5 \ln(z - 4)$

$\ln 8^2 + \ln(z-4)^5 = \boxed{\ln[64(z-4)^5]}$

79. $\ln x - [\ln(x + 1) + \ln(x - 1)]$

$\ln \dfrac{x}{(x+1)(x-1)} = \boxed{\ln \dfrac{x}{x^2-1}}$

80. $4[\ln z + \ln(z + 5)] - 2 \ln(z - 5)$

$\boxed{\ln \dfrac{z^4(z+5)^4}{(z-5)^2}}$

Sketch the following:

(3) $f(x) = 2\ln(3x+6)$ $\quad p(x) = \ln x$

$2\ln(3(x+2))$ $\quad \rightarrow e^y = x$

Transformations: (2) (1) (3)

$x-2$	$\frac{1}{3}x$	x	y	$2(y)$
-1.7	$\frac{1}{3}$	1	0	0
-1.1	$.9$	2.7	1	2

X Y

Domain:
$(-2, \infty)$

Range:
$(-\infty, \infty)$

x-int:
$(-1.7, 0)$

y-int:

Asymptote: $x = -2$

* Horiz compress factor of 1/3
* Shift 2 left
* Vertical stretch factor of 2