Thursday, May 03, 2018
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Name: KEY **Date:** ___________ **Period:** _________

Precalculus Chapter 6 Review

Show ALL WORK on a SEPARATE SHEET OF PAPER.

Solve each triangle. If two solutions exist, find both solutions. If no solution exists, explain why. Round your answers to the nearest hundredth.

1. $A = 41°, a = 15, b = 13$
2. $a = 4, b = 9, c = 10$
3. $B = 150°, a = 10, b = 3$
4. $B = 32°, a = 10, b = 7$

5. Write the complex number in trigonometric form: $5 - 5i$

6. Given $z_1 = 2\sqrt{3} - 2i,\ z_2 = -10i$, complete the following:
   a. write the two complex numbers in trig form
   b. use the trig forms to find $z_1 z_2$ and $\frac{z_1}{z_2}$ where $z_2 \neq 0$.

7. Use DeMoivre's Theorem to find the indicated power of the complex number. Write result in standard form.
   a. $\left[5\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]^4$
   b. $(2+3i)^6$

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Answer Key:

1. $B \approx 34.65°$, $C \approx 104.35°$, $c \approx 22.15$
2. $A \approx 23.56°$, $B \approx 64.06°$, $C \approx 92.39°$
3. No solution
4. Two solutions: $A \approx 49.20°$, $C \approx 98.80°$, $c \approx 13.05$

   $A \approx 130.80°$, $C \approx 17.20°$, $c \approx 3.91$
5. $5\sqrt{2}\left(\cos\frac{[illegible]}{[illegible]} + i\sin\frac{[illegible]}{[illegible]}\right)$
6. (a) [illegible]

   (b) [illegible]
7. (a) $\frac{625}{2} + \frac{[illegible]}{2}i$

   (b) $2035 - 828i$

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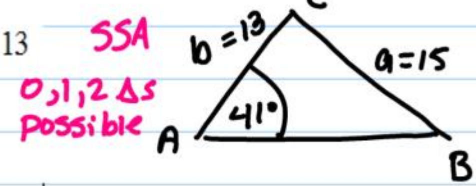
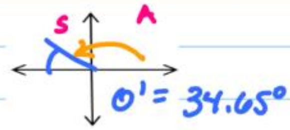
1. $A = 41°, a = 15, b = 13$

SSA

0, 1, 2 Δs possible





1ST Δ:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{15}{\sin 41°} = \frac{13}{\sin B}$$

$$\sin B = \frac{13 \sin 41°}{15}$$ ← pos

$$\sin^{-1}(.5686) = B$$

$$\boxed{B \approx 34.65°}$$ ← STO

$$C = 180° - A - B$$

$$C = 180° - 41° - 34.65°$$

$$\boxed{C \approx 104.35°}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{15}{\sin 41°} = \frac{c}{\sin 104.35°}$$

$$c = \frac{15 \sin 104.35°}{\sin 41°}$$

$$\boxed{c \approx 22.15}$$

2ND Δ:

$$B = 180° - 34.65°$$

$$B = 145.35°$$

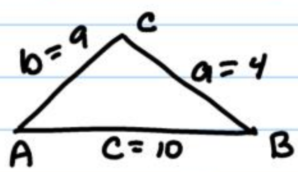
$$C = 180° - A - B$$

$$C = 180° - 41° - 145.35° = \cancel{-6}$$

2ND Δ not possible

$\boxed{\text{one Triangle}}$

2. $a = 4, b = 9, c = 10$



SSS – 1 Δ

① use Law of Cosines to find largest ∠

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(4)^2 + (9)^2 - 10^2}{2(4)(9)}$$

$$\cos C = (-.0417)$$ ← STO

$$\cos^{-1}(-.0417) = C$$

$$\boxed{C \approx 92.39°}$$

② use Law of Sines to find smallest ∠

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{4}{\sin A} = \frac{10}{\sin 92.39°}$$

$$\sin A = \frac{4 \sin 92.39°}{10}$$

$$\sin^{-1}(.3997) = A$$

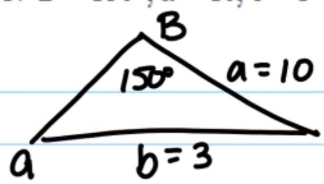
$$\boxed{A \approx 23.56°}$$

③ $B = 180° - A - C$

$$B = 180° - 23.56° - 92.39°$$

$$\boxed{B \approx 64.05°}$$

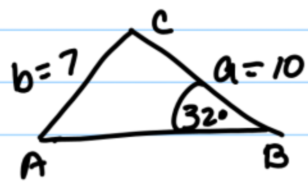
3. $B = 150°, a = 10, b = 3$



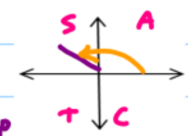
B is largest ∠ since it is obtuse.
Largest side must be opposite largest ∠

$\boxed{\text{NO SOLUTION.}}$

4. $B = 32°, a = 10, b = 7$



SSA: 0, 1, 2 possible Δs



$\theta' = 49.20°$

1ST Δ:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{10}{\sin A} = \frac{7}{\sin 32°}$$

$$\sin A = \frac{10 \sin 32°}{7}$$

pos, STO

$$\sin^{-1}(.7570) = A$$

$$\boxed{A \approx 49.20°}$$

$$C = 180° - A - B$$

$$C = 180° - 49.2° - 32°$$

$$\boxed{C \approx 98.80°}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 98.8°} = \frac{7}{\sin 32°}$$

$$c = \frac{7 \sin 98.8°}{\sin 32°}$$

$$\boxed{c \approx 13.05}$$

2ND Δ

$$A = 180° - 49.20°$$

$$\boxed{A \approx 130.80°}$$

$$C = 180° - A - B$$

$$C = 180° - 130.80° - 32°$$

$$\boxed{C \approx 17.20°}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 17.2°} = \frac{7}{\sin 32°}$$

$$c = \frac{7 \sin 17.2°}{\sin 32°}$$

$$\boxed{c \approx 3.91}$$

5. Write the complex number in trigonometric form: $5 - 5i$

$a = 5$
$b = -5$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{(5)^2 + (-5)^2}$$

$$r = \sqrt{50}$$

$$r = 5\sqrt{2}$$

$$\tan\theta' = \frac{b}{a}$$

$$\tan\theta' = \frac{5}{-5} = -1$$

$\theta' = \frac{\pi}{4}$ in Q IV $\theta = \frac{7\pi}{4}$

$$z = r(\cos\theta + i\sin\theta)$$

$$\boxed{z = 5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$$

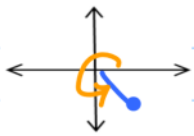
6. Given $z_1 = 2\sqrt{3} - 2i,\ z_2 = -10i$, complete the following:

a. write the two complex numbers in trig form

a) $z_1 = 2\sqrt{3} - 2i$

$a = 2\sqrt{3}$

$b = -2$



$r = \sqrt{a^2 + b^2}$

$r = \sqrt{(2\sqrt{3})^2 + (-2)^2}$

$r = \sqrt{16}$

$r = 4$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$

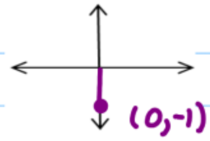
$\theta' \approx \frac{\pi}{6}$

in Q IV $\theta = \frac{11\pi}{6}$

$z_2 = -10i$

$a = 0$

$b = -10$



$r = \sqrt{a^2 + b^2}$

$r = \sqrt{0^2 + (-10)^2}$

$r = \sqrt{100}$

$r = 10$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{-10}{0}$

= undef

$\theta = \frac{3\pi}{2}$

$z = r(\cos\theta + i\sin\theta)$

$\boxed{z = 4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}$

$\boxed{z = 10\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)}$

b. use the trig forms to find $z_1 z_2$ and $\frac{z_1}{z_2}$ where $z_2 \neq 0$.

$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$= 4 \cdot 10 \left[\cos\left(\frac{11\pi}{6} + \frac{3\pi}{2}\cdot\frac{3}{3}\right) + i\sin\left(\frac{11\pi}{6} + \frac{3\pi}{2}\cdot\frac{3}{3}\right)\right.$

$= 40\left(\cos\frac{20\pi}{6} + i\sin\frac{20\pi}{6}\right)$ * we restrict θ $0 \le \theta < 2\pi$

$= \boxed{40\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)}$ * subtract mults of 2π

$\frac{20\pi}{6} - \frac{12\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

$= \frac{4}{10}\left[\cos\left(\frac{11\pi}{6} - \frac{3\pi}{2}\cdot\frac{3}{3}\right) + i\sin\left(\frac{11\pi}{6} - \frac{3\pi}{2}\cdot\frac{3}{3}\right)\right]$

$= \boxed{\frac{2}{5}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

7. Use DeMoivre's Theorem to find the indicated power of the complex number. Write result in standard form.

a. $\left[5\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)\right]^4$ $\quad r=5$, $n=4$

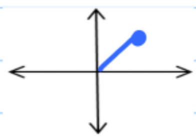
$$z = r^n(\cos n\theta + i\sin n\theta)$$

$$z^4 = 5^4\left(\cos 4\cdot\tfrac{\pi}{12} + i\sin 4\cdot\tfrac{\pi}{12}\right)$$

$$= 625\left(\cos\tfrac{\pi}{3} + i\sin\tfrac{\pi}{3}\right)$$

$$= 625\left(\tfrac{1}{2} + i\tfrac{\sqrt{3}}{2}\right) = \boxed{\tfrac{625}{2} + \tfrac{625\sqrt{3}}{2}i}$$

b. $(2+3i)^6$ $\quad a=2$, $b=3$



$r=\sqrt{a^2+b^2}$ $\quad \tan\theta' = \frac{b}{a}$ $\quad z=r(\cos\theta+i\sin\theta)$

$r=\sqrt{2^2+3^2}$ $\quad \tan\theta' = \frac{3}{2}$ $\quad \boxed{z=\sqrt{13}(\cos .9828 + i\sin .9828)}$

$r=\sqrt{13}$ $\quad \theta' = .9828 \leftarrow$ STO

in QI $\theta = .9828$

$$z^n = r^n(\cos n\theta + i\sin n\theta) \qquad n=6 \quad r=\sqrt{13}$$

$$z^6 = (\sqrt{13})^6\left(\cos 6(.9828) + i\sin 6(.9828)\right)$$

STO

$$= 2197(.9263 - .3769i)$$

$$= \boxed{2035 - 828i}$$