Monday, March 19, 2018
6:49 PM

Precalculus
Ch.5 Test Review #3

Name: KEY
Date: ________________ Period: ______

1. Given that angle A is in Q III with $\sin A = -\frac{40}{41}$, angle B is in Q IV with $\tan B = -\frac{5}{4}$, angle C is in Q II with $\csc C = \frac{13}{12}$, find each of the following. Assume each angle is in the interval $[0, 2\pi)$

a) $\cos(A+B)$ $\boxed{\frac{-236\sqrt{41}}{1681}}$

b) $\sin 2B$ $\boxed{-\frac{40}{41}}$

c) $\cos\frac{C}{2}$ $\boxed{\frac{2\sqrt{13}}{13}}$

d) $\tan 2C$ $\boxed{\frac{120}{119}}$

e) $\sin\frac{1}{2}A$ $\boxed{\frac{5\sqrt{41}}{41}}$

f) $\sin(B-C)$ $\boxed{\frac{-23\sqrt{41}}{533}}$

g) $\tan\frac{1}{2}B$ $\boxed{\frac{\sqrt{41}-4}{-5}}$

h) $\cos 2A$ $\boxed{\frac{-1519}{1681}}$

2. Find all solutions in the interval $[0, 2\pi)$ for:

a) $\cos 2x = 11\cos x + 5$

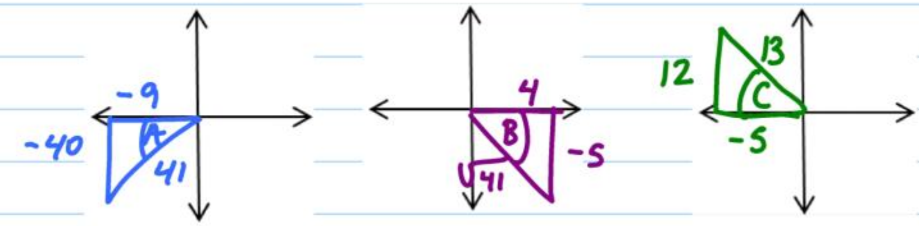
$\boxed{x = \frac{2\pi}{3}}$ $\boxed{x = \frac{4\pi}{3}}$

b) $3\sin x = \cos 2x + 1$

$\boxed{x = \frac{\pi}{6}}$ $\boxed{x = \frac{5\pi}{6}}$

$\frac{h}{o}$

1. Given that angle A is in Q III with $\sin A = -\frac{40}{41}$, angle B is in Q IV with $\tan B = -\frac{5}{4}$, angle C is in Q II with $\csc C = \frac{13}{12}$, find each of the following. Assume each angle is in the interval $[0,2\pi)$



a) $\cos(A+B)$ $= \cos A \cos B - \sin A \sin B$

$= \left(\frac{-9}{41}\right)\left(\frac{4}{\sqrt{41}}\right) - \left(-\frac{40}{41}\right)\left(\frac{-5}{\sqrt{41}}\right) = \frac{-36}{41\sqrt{41}} - \frac{200}{41\sqrt{41}}$

$= \frac{-236}{41\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \boxed{\frac{-236\sqrt{41}}{1681}}$

b) $\sin 2B$ $= 2\sin B \cos B$

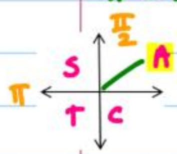
$= 2\left(\frac{-5}{\sqrt{41}}\right)\left(\frac{4}{\sqrt{41}}\right) = \boxed{\frac{-40}{41}}$

c) $\cos\frac{C}{2}$ $= + \sqrt{\frac{1+\cos C}{2}} = + \sqrt{\frac{1+\frac{-5}{13}}{2}} = \sqrt{\frac{\frac{8}{13}}{2}} = \sqrt{\frac{8}{13} \cdot \frac{1}{2}}$

$= \sqrt{\frac{\cancel{8}\,4}{\cancel{26}\,13}} = \frac{\sqrt{4}}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$= \boxed{\frac{2\sqrt{13}}{13}}$

* It is given that C is in Q II



$\frac{1}{2}\left(\frac{\pi}{2}\right) < \frac{C}{2} < \frac{\pi}{2}$

$\frac{\pi}{4} < \frac{C}{2} < \frac{\pi}{2}$

* so $\frac{C}{2}$ is in Q I

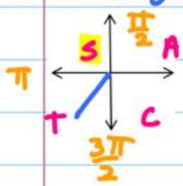
* cosine pos in Q I

d) $\tan 2C$ $= \frac{2\tan C}{1-\tan^2 C} = \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{\frac{-24}{5}}{1-\frac{144}{25}} = \frac{\frac{-24}{5}}{\frac{-119}{25}}$

$= \frac{-24}{\cancel{5}_1} \cdot \frac{\cancel{25}^{5}}{-119} = \frac{-120}{-119} = \boxed{\frac{120}{119}}$

e) $\sin\frac{1}{2}A = +\sqrt{\frac{1-\cos A}{2}} = +\sqrt{\frac{1-\frac{-9}{41}}{2}} = +\sqrt{\frac{\frac{50}{41}}{2}} = \sqrt{\frac{\cancel{50}^{25}}{41}\cdot\frac{1}{\cancel{2}_{1}}}$

* It is given that A is in Q III



$\frac{\pi}{2} < \frac{A}{2} < \left(\frac{3\pi}{2}\right)\frac{1}{2}$

$\frac{\pi}{2} < \frac{A}{2} < \frac{3\pi}{4}$

* so $\frac{A}{2}$ is in Q 2

* Sine pos in Q2

$= \frac{\sqrt{25}}{\sqrt{41}} = \frac{5}{\sqrt{41}}\cdot\frac{\sqrt{41}}{\sqrt{41}} = \boxed{\frac{5\sqrt{41}}{41}}$

f) $\sin(B-C) = \sin B\cos C - \cos B\sin C$

$= \left(\frac{-5}{\sqrt{41}}\right)\left(\frac{-5}{13}\right) - \left(\frac{4}{\sqrt{41}}\right)\left(\frac{12}{13}\right) = \frac{25}{13\sqrt{41}} - \frac{48}{13\sqrt{41}}$

$= \frac{-23}{13\sqrt{41}}\cdot\frac{\sqrt{41}}{\sqrt{41}} = \boxed{\frac{-23\sqrt{41}}{533}}$

g) $\tan\frac{1}{2}B = \frac{1-\cos B}{\sin B} = \frac{1-\frac{4}{\sqrt{41}}}{\frac{-5}{\sqrt{41}}} = \frac{\frac{\sqrt{41}-4}{\sqrt{41}}}{\frac{-5}{\sqrt{41}}} = \frac{\sqrt{41}-4}{\cancel{\sqrt{41}}}\cdot\frac{\cancel{\sqrt{41}}}{-5}$

$= \boxed{\frac{\sqrt{41}-4}{-5}}$

h) $\cos 2A = 2\cos^2 A - 1$

$= 2\left(\frac{-9}{41}\right)^2 - 1 = 2\left(\frac{81}{1681}\right) - 1 = \frac{162}{1681} - \frac{1681}{1681}$

$= \boxed{\frac{-1519}{1681}}$

2. Find all solutions in the interval $[0, 2\pi)$ for:

a) $\cos 2x = 11\cos x + 5$ * Double Angle Formula

$2\cos^2 x - 1 = 11\cos x + 5$

$2\cos^2 x - 1 - 11\cos x - 5 = 0$

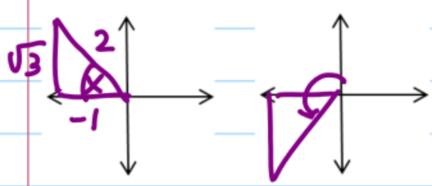
$2\cos^2 x - 11\cos x - 6 = 0$ $2x^2 - 11x - 6$

$(2\cos x + 1)(\cos x - 6) = 0$ $(2x+1)(x-6)$

$2\cos x + 1 = 0$ $\cos x - 6 = 0$

S A / T C $\cos x = -\frac{1}{2}$ $\cos x = 6$

no solution



$\theta' = \frac{\pi}{3}$

$\boxed{x = \frac{2\pi}{3}}$ $\boxed{x = \frac{4\pi}{3}}$

b) $3\sin x = \cos 2x + 1$ * Double-Angle formula

$3\sin x = 1 - 2\sin^2 x + 1$

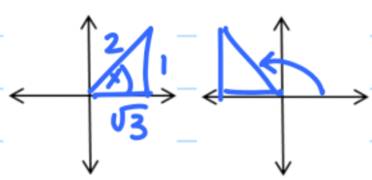
$2\sin^2 x + 3\sin x - 2 = 0$ $2x^2 + 3x - 2$

$(2\sin x - 1)(\sin x + 2) = 0$ $(2x-1)(x+2)$

$2\sin x - 1 = 0$ $\sin x + 2 = 0$

S A / T C $\sin x = \frac{1}{2}$ $\sin x = -2$

no solution



$\boxed{x = \frac{\pi}{6}}$ $\boxed{x = \frac{5\pi}{6}}$