Saturday, March 17, 2018
9:31 AM

Precalculus
Ch.5 Test Review #2

Name: KEY
Date:__________ Period:______

SHOW ALL WORK on a separate piece of paper, which will be collected at the end of the period. Credit will be given for WORK SHOWN.

1. Which is a trigonometric identity?

   a) $\sec u = \dfrac{1}{\cos u}$ b) $\sin\left(\dfrac{\pi}{2} + u\right) = \sin u$

   c) $\tan^2 u + \cot^2 u = 1$ d) $\sec u = \sin \dfrac{1}{u}$

2. Factor the expression and use the fundamental identities to simplify $\cos^2 x \sec^2 x - \cos^2 x$

   a) $\cos^2 x \cot^2 x$ b) $\cos^2 x$

   c) 1 d) $\sin^2 x$

   $= \cos^2 x(\sec^2 x - 1)$
   $= \cos^2 x(\tan^2 x)$
   $= \cos^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x}$

   $= \sin^2 x$

For #3 – 4, identify the expression that completes the equation so that it is an identity.

3. $\dfrac{(1+\sec u)}{(1+\sec u)} \cdot \dfrac{1+\sec u}{\tan u} - \dfrac{\tan u}{1+\sec u} \cdot \dfrac{\tan u}{\tan u} =$

   a) 0 b) $2\sin u$

   c) $2 + \cos u$ d) $2\cot u$

   $\dfrac{1 + 2\sec u + \underbrace{\sec^2 u - \tan^2 u}}{(1+\sec u)\tan u} = \dfrac{2 + 2\sec u}{(1+\sec u)\tan u}$

   $= \dfrac{2(1+\cancel{\sec u})}{(1+\cancel{\sec u})\tan u}$

   $= 2\cot u$

4. $\dfrac{\cos x}{1+\sin x} = \dfrac{(1-\sin x)}{(1-\sin x)}$

   a) $\dfrac{1-\sin x}{\cos x}$ b) $\sin x$

   c) $\dfrac{1+\sec x}{\sec x}$ d) $\dfrac{1+\csc x}{\csc x}$

   $= \dfrac{\cos x(1-\sin x)}{1-\sin^2 x} = \dfrac{\cos x(1-\sin x)}{\cos^2 x}$

   $= \dfrac{1-\sin x}{\cos x}$

For #5 – 6, identify the x-values that are solutions of the equation.

5. $5\sqrt{3}\tan x + 3 = 8\sqrt{3}\tan x$

a) $x = \frac{\pi}{6}, x = \frac{11\pi}{6}$

b) $x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$

c) $x = \frac{\pi}{6}, x = \frac{7\pi}{6}$

d) $x = \frac{5\pi}{6}, x = \frac{11\pi}{6}$

$3 = 3\sqrt{3}\tan x$

$1 = \sqrt{3}\tan x$

$\frac{1}{\sqrt{3}} = \tan x$

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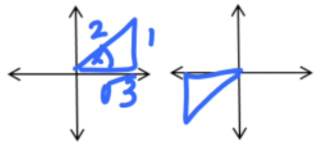


$x = \frac{\pi}{6}$ $\quad \frac{7\pi}{6}$

6. $3\cot^2 x - 9 = 0$

a) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

b) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

c) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

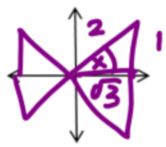
d) $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}\right\}$

$3\cot^2 x = 9$

$\cot^2 x = 3$

$\cot x = \pm\sqrt{3}$

$\frac{a}{o}$



$x = \frac{\pi}{6}$

For #7 – 8, find all solutions of the equation in the interval $[0, 2\pi)$.

7. $\sec^2 x + \tan x = 1$

a) $\left\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}\right\}$

b) $\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

c) $\{0\}$

d) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

$\underline{1 + \tan^2 x} + \tan x = 1$

$\tan^2 x + \tan x + 1 = 1$

$\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\frac{y}{x}$ $\quad \tan x = 0 \qquad \tan x = -1$

S | A
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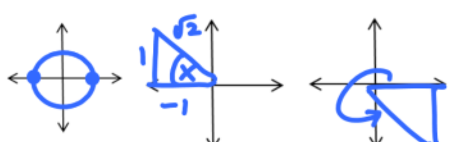


$x = 0, \pi \qquad x = \frac{3\pi}{4} \qquad x = \frac{7\pi}{4}$

8. $2\csc 3x - \frac{4}{3}\sqrt{3} = 0$

~~a)~~ $\left\{\frac{\pi}{9}, \frac{7\pi}{9}\right\}$

b) $\left\{\frac{\pi}{9}, \frac{2\pi}{9}, \frac{\pi}{2}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}\right\}$

c) $\left\{\frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}\right\}$

d) $\left\{\frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}\right\}$

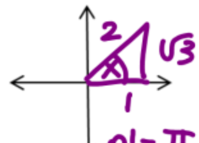
$\frac{1}{2}(2\csc 3x) = \left(\frac{4}{3}\sqrt{3}\right)\frac{1}{2}$

$\csc 3x = \frac{4\sqrt{3}}{6}$

$\csc 3x = \frac{2\sqrt{3}}{3}$

S | A
T | C

$\sin 3x = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

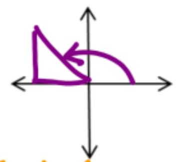


$\theta' = \frac{\pi}{3}$

$\frac{1}{3}(3x) = \left(\frac{\pi}{3} + 2\pi n\right)\frac{1}{3}$

$x = \frac{\pi}{9} + \frac{2\pi}{3}$

$\frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}$



$\frac{1}{3}(3x) = \left(\frac{2\pi}{3} + 2\pi n\right)\frac{1}{3}$

$x = \frac{2\pi}{9} + \frac{2\pi}{3}\cdot\frac{3}{3}$ $\quad \frac{6\pi}{9}$

$\frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}$

9. $12\sin^2 x - 4\cos x - 11 = 0$

a) $\left\{\arccos\frac{1}{6} = 1.4033, 2\pi - \arccos\frac{1}{6} = 4.8799, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

b) $\left\{\arccos(-6), \arccos(-6) + \pi, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

c) $\left\{\arccos(-6), \arccos(-6) + \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

d) $\left\{\arccos\frac{1}{6} = 1.4033, 2\pi - \arccos\frac{1}{6} = 4.8799, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

$12(1 - \cos^2 x) - 4\cos x - 11 = 0$

$12 - 12\cos^2 x - 4\cos x - 11 = 0$

$-12\cos^2 x - 4\cos x + 1 = 0$

$-(12\cos^2 x + 4\cos x - 1) = 0$ $\quad 12x^2 + 4x - 1$

$-(6\cos x - 1)(2\cos x + 1) = 0$ $\quad (6x-1)(2x+1)$

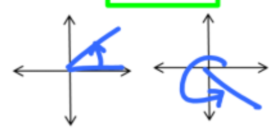
S | A
T | C

$6\cos x - 1 = 0$

$\cos x = \frac{1}{6}$

$\cos^{-1}\left(\frac{1}{6}\right) = x$

$x = \boxed{1.4033}$



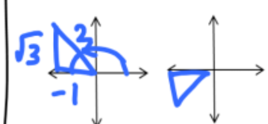
$2\pi - 1.4033$

$= \boxed{4.8799}$

S | A
T | C

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$



$\theta' = \frac{\pi}{3}$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

10. Find the exact value of $\cos 285^\circ$.

a) $\frac{-\sqrt{6}+\sqrt{2}}{4}$

b) $\frac{\sqrt{6}+\sqrt{2}}{4}$

c) $\frac{-\sqrt{2}+\sqrt{6}}{4}$

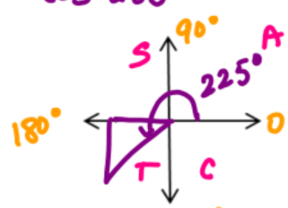
d) $\frac{-\sqrt{2}-\sqrt{6}}{4}$

$\cos 285^\circ = \cos(60^\circ + 225^\circ) = \cos 60^\circ \cos 225^\circ - \sin 60^\circ \sin 225^\circ$

$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{-\sqrt{2}+\sqrt{6}}{4}$



$\theta' = 45^\circ$

11. Identify the expression that completes the identity $\cos\left(\frac{\pi}{4}-x\right)=$

a) $\frac{\sqrt{2}}{2}(\cos x-\sin x)$ b) $\frac{\sqrt{2}}{2}(\sin x-\cos x)$

c) $-\frac{\sqrt{2}}{2}(\cos x+\sin x)$ d) $\frac{\sqrt{2}}{2}(\cos x+\sin x)$

* Difference Formula

$= \cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x$

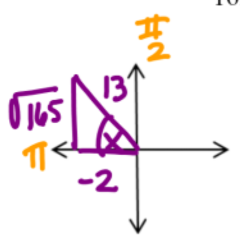
$= \left(\frac{\sqrt{2}}{2}\right)\cos x + \left(\frac{\sqrt{2}}{2}\right)\sin x$

$= \left(\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)$

12. Find the exact value of $\cos 2x$ using the double angle formula.

$\cos x = -\frac{2}{13};\quad \frac{\pi}{2}<x<\pi$

a) $-\frac{161}{169}$ b) $\frac{4\sqrt{165}}{169}$ c) $\frac{161}{169}$ d) $-\frac{4\sqrt{165}}{169}$



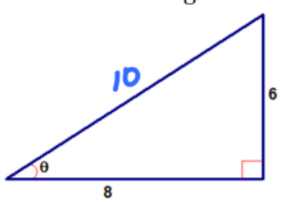
$\cos 2x = 2\cos^2 x - 1$

$= 2\left(-\frac{2}{13}\right)^2 - 1$

$= 2\left(\frac{4}{169}\right) - 1$

$= \frac{8}{169} - 1 \quad = \frac{8}{169} - \frac{169}{169} = \frac{-161}{169}$

13. Use the figure to find the exact value of the trigonometric function $\cot\frac{\theta}{2}$



a) $\frac{\sqrt{10}}{10}$ b) $\sqrt{10}$ c) $\frac{1}{3}$ d) 3

$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$

* Use Half-Angle formula for tan
* then find the reciprocal

$\tan\frac{u}{2} = \frac{1-\frac{8}{10}}{\frac{6}{10}} = \frac{\frac{2}{10}}{\frac{6}{10}} = \frac{2}{10}\cdot\frac{10}{6} = \frac{1}{3}$

$\cot\frac{u}{2} = \frac{3}{1} = 3$ (Reciprocal of tan)

ANSWERS:	1. A	2. D	3. D	4. A	5. C	6. C
7. A	8. D	9. A	10. C	11. D	12. A	13. D