Tuesday, April 30, 2019
6:35 PM

KEY

6.5C - Trig Form of a Complex Number

Homework:
- pg 478-479 (VC #1-3) # 49,57,61,73,81
- Test ~~Friday~~ next week on 2.4, 6.1,6.2,6.5

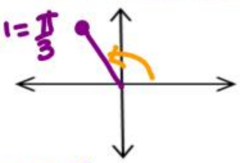
Objective:

SWBAT: review working with the trig form ofcomplex numbers

Do Now:

Use DeMoivre's Theorem to find: $\left(-1+\sqrt{3}i\right)^{12}$

$\theta' = \frac{\pi}{3}$



$a = -1$
$b = \sqrt{3}$

> **DeMoivre's Theorem**
>
> If $z = r(\cos\theta + i\sin\theta)$ is a complex numbers and n is a positive integer then,
>
> $z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$

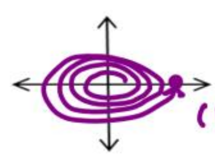
$r = \sqrt{a^2+b^2}$
$r = \sqrt{(-1)^2+(\sqrt{3})^2}$
$r = \sqrt{4}$
$r = 2$

$\tan\theta' = \frac{b}{a}$
$\tan\theta' = \frac{\sqrt{3}}{-1} = -\sqrt{3}$
$\theta' = \frac{\pi}{3}$
in QII $\theta = \frac{2\pi}{3}$

TRIG FORM: $z = r(\cos\theta + i\sin\theta)$
$z = 2\left(\cos\frac{2\pi}{3} + i\ \frac{2\pi}{3}\right)$

$z = r^n(\cos n\theta + i\sin n\theta)$ $r=2$ $n=12$

$z^{12} = 2^{12}\left(\cos\left(12\cdot\frac{2\pi}{3}\right) + i\sin\left(12\cdot\frac{2\pi}{3}\right)\right)$
$= 4096(\cos 8\pi + i\sin 8\pi)$
$= 4096(\cos 0 + i\sin 0)$
$= 4096(1 + i(0))$
$= \boxed{4096}$



(1,0)

Classwork...

Ex 1: Use DeMoivre's Theorem to find: $(2+2i)^6$

Ex 2: Convert the following to trig form and find the product. Check your answer by finding the product in standard form.

$$z_1 = -1+\sqrt{3}i \quad \text{and} \quad z_2 = 4\sqrt{3}-4i$$

Ex 3: Find the indicated power of the complex number.

$$\left[2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\right]^8$$

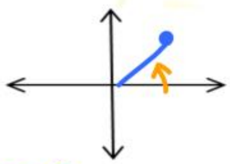
(a) Write the trig forms of the complex numbers
(b) Perform the indicated operation using the trig form
(c) Perform the indicated operations using the standard form and check your results with part (b)

Ex 4: $4\left(1-\sqrt{3}i\right)$

Ex 5: $\dfrac{4i}{-2+2i}$

Practice makes perfect!

Ex 1: Use DeMoivre's Theorem to find: $(2+2i)^6$



$a = 2$
$b = 2$

$r = \sqrt{a^2+b^2}$
$r = \sqrt{2^2+2^2}$
$r = \sqrt{8}$
$r = 2\sqrt{2}$

$\tan\theta' = \frac{b}{a}$
$\tan\theta' = \frac{2}{2} = 1$
$\theta' = \frac{\pi}{4}$
in QI $\theta = \frac{\pi}{4}$

TRIG FORM: $z = r(\cos\theta + i\sin\theta)$
$z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

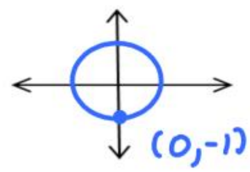
$z = r^n(\cos n\theta + i\sin n\theta)$ $\quad r = 2\sqrt{2}$ $\quad n = 6$

$z^6 = (2\sqrt{2})^6\left[\cos\left(6\cdot\frac{\pi}{4}\right) + i\sin\left(6\cdot\frac{\pi}{4}\right)\right]$

$= 512\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$

$= 512[0 + i(-1)]$

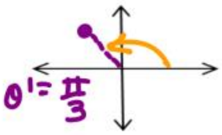
$= \boxed{-512i}$



Ex 2: Convert the following to trig form and find the product. Check your answer by finding the product in standard form.

$$z_1 = -1 + \sqrt{3}i \quad \text{and} \quad z_2 = 4\sqrt{3} - 4i$$

z_1: $a = -1$, $b = \sqrt{3}$



$\theta' = \frac{\pi}{3}$

$r = \sqrt{a^2+b^2}$

$r = \sqrt{(-1)^2 + (\sqrt{3})^2}$

$r = \sqrt{4}$

$r = 2$

$\tan\theta' = \frac{b}{a}$

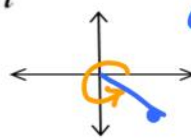
$\tan\theta' = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\theta' = \frac{\pi}{3}$

in QII

$\theta = \frac{2\pi}{3}$

z_2: $a = 4\sqrt{3}$, $b = -4$



$\theta' = \frac{\pi}{6}$

$r = \sqrt{a^2+b^2}$

$r = \sqrt{(4\sqrt{3})^2 + (-4)^2}$

$r = \sqrt{64}$

$r = 8$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{-4}{4\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\theta' = \frac{\pi}{6}$

in QIV

$\theta = \frac{11\pi}{6}$

TRIG FORM: $z = r(\cos\theta + i\sin\theta)$

$\boxed{z_1 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)}$ $\boxed{z_2 = 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}$

Product: $z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$

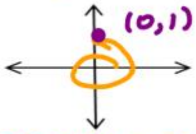
$= 2 \cdot 8 \left[\cos\left(\frac{2\pi}{3}\cdot\frac{2}{2} + \frac{11\pi}{6}\right) + i\sin\left(\frac{2\pi}{3}\cdot\frac{2}{2} + \frac{11\pi}{6}\right)\right]$

$= 16\left[\cos\frac{15\pi}{6} + i\sin\frac{15\pi}{6}\right]$ *

$= \boxed{16\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)} = 16(0 + i(1)) = \boxed{16i}$

TRIG FORM — Std FORM

* we restrict θ: $0 \le \theta < 2\pi$



$\frac{15\pi}{6} - \frac{12\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

Check: $(-1 + \sqrt{3}i)(4\sqrt{3} - 4i)$

$= -4\sqrt{3} + 4i + 4 \cdot 3i - 4\sqrt{3}i^2$

$= -4\sqrt{3} + 16i + 4\sqrt{3}$

$= \boxed{16i}$ ✓ * matches answer in std form

DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex numbers and n is a positive integer then,

$$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Ex 3: Find the indicated power of the complex number.

$$\left[2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]^8$$

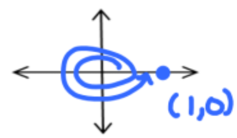
$n = 8$

$r = 2$

$z^n = r^n(\cos n\theta + i\sin n\theta)$

$z^8 = 2^8\left[\cos\left(8\cdot\frac{\pi}{2}\right) + i\sin\left(8\cdot\frac{\pi}{2}\right)\right]$

$= 256\left[\cos 4\pi + i\sin 4\pi\right]$

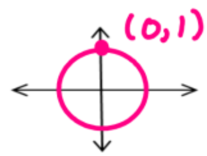


$= 256(1 + i(0))$

$= \boxed{256}$

How can you check your answer?

Check: $\left[2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]^8$



$= [2(0 + i(1))]^8$

$= (2i)^8$

* evaluate + simplify

$= 2^8 i^8$

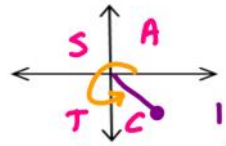
$= 256(1)(1)$

$= 256$ ✓

The fun never ends...

(a) Write the trig forms of the complex numbers
(b) Perform the indicated operation using the trig form
(c) Perform the indicated operations using the standard forms and check your results with part (b)

Ex. 4: $4\left(1-\sqrt{3}i\right)$ $\quad a=1$, $b=-\sqrt{3}$



$1-\sqrt{3}i$

$\theta' = \frac{\pi}{3}$

a)

$r=\sqrt{a^2+b^2}$

$r=\sqrt{1^2+(-\sqrt{3})^2}$

$r=\sqrt{4}$

$r=2$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

$\theta' = \frac{\pi}{3}$

QIV $\theta = \frac{5\pi}{3}$

$z = r(\cos\theta + i\sin\theta)$

$= \boxed{2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)}$

TRIG FORM

b) $4\cdot 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$= 8\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) \quad = \boxed{4-4\sqrt{3}i}$

c) $4(1-\sqrt{3}i) = \boxed{4-4\sqrt{3}i}$ $\quad$ * Answers match ✓

The fun never ends...

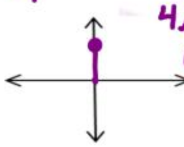
(a) Write the trig forms of the complex numbers
(b) Perform the indicated operation using the trig form
(c) Perform the indicated operations using the standard forms and check your results with part (b)

a)

Ex. 5: $\dfrac{4i}{-2+2i} = \dfrac{z_1}{z_2}$

z_1

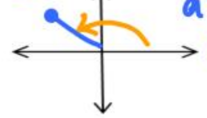
$4i$



$a=0$
$b=4$

z_2

$-2+2i$
$a=-2$
$b=2$



$r=\sqrt{a^2+b^2}$
$r=\sqrt{(0)^2+(4)^2}$
$r=\sqrt{16}$
$r=4$

$\tan\theta' = \frac{b}{a}$
$\tan\theta' = \frac{4}{0}$ $\frac{y}{x}$
$\theta' = \text{undef}$
$\theta = \frac{\pi}{2}$

$r=\sqrt{a^2+b^2}$
$r=\sqrt{(-2)^2+(2)^2}$
$r=\sqrt{8}$
$r=2\sqrt{2}$

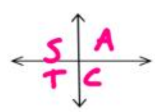
$\tan\theta' = \frac{b}{a}$
$\tan\theta' = \frac{2}{-2} = -1$
$\theta' = \frac{\pi}{4}$
in QII
$\theta = \frac{3\pi}{4}$

TRIG FORM: $z = r(\cos\theta + i\sin\theta)$

$\boxed{z_1 = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$ $\boxed{z_2 = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}$

b) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$

$= \frac{\sqrt{2}}{\sqrt{2}}\frac{4}{2\sqrt{2}}\left[\cos\left(\frac{\pi}{2}\cdot\frac{2}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{2}\cdot\frac{2}{2} - \frac{3\pi}{4}\right)\right]$



$= \boxed{\sqrt{2}\left(\cos-\frac{\pi}{4} + i\sin-\frac{\pi}{4}\right)} = \sqrt{2}\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$

How can you check your answer? $= \boxed{1-i}$

c) $\dfrac{4i}{(-2+2i)} \cdot \dfrac{(-2-2i)}{(-2-2i)} = \dfrac{-8i-8i^2}{4+4i-4i-4i^2} = \dfrac{-8i-8(-1)}{4-4(-1)}$

$= \dfrac{-8i+8}{8} = -i+1 = \boxed{1-i}$