Monday, April 29, 2019
5:24 PM

KEY

6.5B - Trig Form of a Complex Number

Homework:
- pg 478-479 #15,21,27,47,53,59,63,79,87
- Test Friday on 2.4, 6.1,6.2,6.5

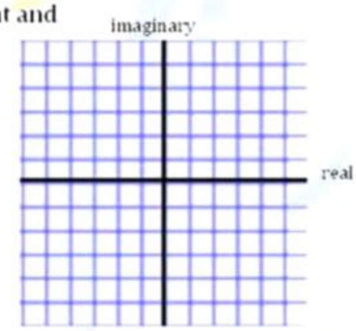
Objective:

SWBAT: Multiply and Divide complex numbers in trig form

Do Now: Take and complete the *half sheet* from the front desk.

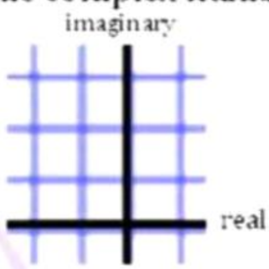
Do Now

1. Plot the complex number on the graph to the right and find its absolute value: $5 - 12i$



2. Write the complex number in trig form:

$z = -1 + \sqrt{3}i$



Do Now

3. Find the trig form of

   a. $4 - 4\sqrt{3}i$ b. $-7 + 4i$

4. Write in trig form: $6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

Multiplying/Dividing Complex Numbers in Trig Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$.

Product Rule:

$$z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

Quotient Rule:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

☺YES! You have to memorize these formulas.☺

Example 1

$$z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

Find the product of the following complex numbers and write the result in both trig and standard forms.

$r_1 = 3$

$r_2 = 4$

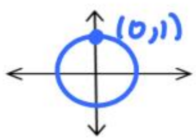
$$z_1 = 3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) \ \ and \ \ z_2 = 4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$$

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

$$z_1 z_2 = 3 \cdot 4 \left[\cos\left(\frac{\pi}{3}\cdot\frac{2}{2} + \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3}\cdot\frac{2}{2} + \frac{\pi}{6}\right)\right]$$

$$= \boxed{12\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]}$$ TRIG FORM

* evaluate



$$z_1 z_2 = 12\left[0 + i(1)\right]$$

$$= \boxed{12i}$$ STANDARD FORM

Example 2

$$z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

Find the quotient of the following complex numbers and write the result in both trig and standard forms.

$r_1 = 1$
$r_2 = 1$

$$z_1 = \cos 40° + i\sin 40° \quad and \quad z_2 = \cos 10° + i\sin 10°$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

$$= \frac{1}{1}\left[\cos(40° - 10°) + i\sin(40° - 10°)\right]$$

$$= \boxed{\cos 30° + i\sin 30°}$$ TRIG FORM

evaluate

$$\frac{z_1}{z_2} = \frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)$$

$$= \boxed{\frac{\sqrt{3}}{2} + \frac{1}{2}i}$$ standard form

Example 3

$$z_1 \cdot z_2 = r_1 \cdot r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

Perform the operation and leave the result in trig form.

$$\frac{6(\cos 40° + i\sin 40°)}{7(\cos 100° + i\sin 100°)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

$$= \frac{6}{7}[\cos(40° - 100°) + i\sin(40° - 100°)]$$

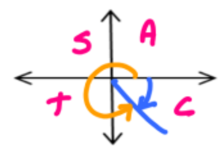
$$= \frac{6}{7}[\cos(-60°) + i\sin(-60°)]$$

* we restrict θ $0 \le \theta < 2\pi$

$$= \boxed{\frac{6}{7}(\cos 300° + i\sin 300°)}$$

TRIG FORM

evaluate



$\theta' = 60°$

$\theta = 360° - 60°$

$\theta = 300°$

$$\frac{z_1}{z_2} = \frac{\overset{3}{\cancel{6}}}{7}\left(\frac{1}{\cancel{2}_1} + i\left(-\frac{\sqrt{3}}{\cancel{2}_1}\right)\right)$$

$$= \boxed{\frac{3}{7} + -\frac{3\sqrt{3}}{7}i}$$

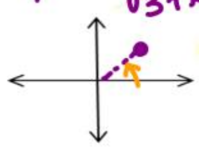
standard form

Example 4

$a=\sqrt{3}$ $b=1$ $a=1$ $b=1$

Given that $z_1 = \sqrt{3}+i$ and $z_2 = 1+i$, write in trig form & find:

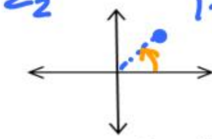
z_1 $\sqrt{3}+i$



1. $\dfrac{z_1}{z_2}$

2. $z_1 z_2$

z_2 $1+i$



$r=\sqrt{a^2+b^2}$

$r=\sqrt{(\sqrt{3})^2+(1)^2}$

$r=\sqrt{4}$

$r=2$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{1}{\sqrt{3}}$

$\theta' = \frac{\pi}{6}$

in QI

$\theta = \frac{\pi}{6}$

$r=\sqrt{a^2+b^2}$

$r=\sqrt{(1)^2+(1)^2}$

$r=\sqrt{2}$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{1}{1} = 1$

$\theta' = \frac{\pi}{4}$

in QI

$\theta = \frac{\pi}{4}$

TRIG FORM: $z = r(\cos\theta + i\sin\theta)$

$\boxed{z_1 = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})}$ $\boxed{z_2 = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})}$

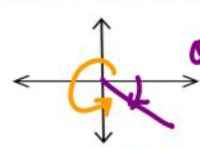
1) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$

$= \dfrac{2}{\sqrt{2}}\left[\cos\left(\frac{\pi}{6}\cdot\frac{2}{2} - \frac{\pi}{4}\cdot\frac{3}{3}\right) + i\sin\left(\frac{\pi}{6}\cdot\frac{2}{2} - \frac{\pi}{4}\cdot\frac{3}{3}\right)\right.$

$= \dfrac{\sqrt{2}}{\sqrt{2}}\dfrac{2}{\sqrt{2}}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$

* we restrict θ $0 \le \theta < 2\pi$

$= \boxed{\sqrt{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)}$



$\theta' = \frac{\pi}{12}$

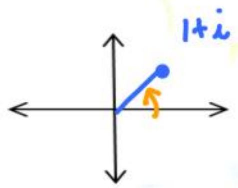
$\theta = \frac{23\pi}{12}$

2) $z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$

$= 2\sqrt{2}\left[\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)\right]$

$= \boxed{2\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)}$

Finding Powers of Complex Numbers



DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex numbers and n is a positive integer then,

$$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$

$a=1$
$b=1$

EX: 5 Use DeMoivre's Theorem to find: $(1+i)^6$

$r = \sqrt{a^2+b^2}$
$r = \sqrt{1^2+1^2}$
$r = \sqrt{2}$

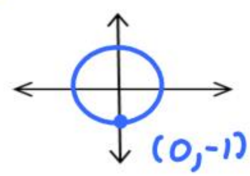
$\tan\theta' = \frac{b}{a}$
$\tan\theta' = \frac{1}{1} = 1$
$\theta' = \frac{\pi}{4}$
in QI $\theta = \frac{\pi}{4}$

TRIG FORM: $z = r(\cos\theta + i\sin\theta)$
$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z^n = [r(\cos\theta + i\sin\theta)]^n$ $r = \sqrt{2}$ $n = 6$
$z = r^n(\cos n\theta + i\sin n\theta)$
$z^6 = (\sqrt{2})^6\left(\cos\left(6\cdot\frac{\pi}{4}\right) + i\sin\left(6\cdot\frac{\pi}{4}\right)\right)$

Memorize this one too!

$= 8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$



$= 8(0 + -1i)$
$= \boxed{-8i}$