Sunday, April 28, 2019
4:28 PM

KEY

6.5A - Trig Form of a Complex Number

Homework:
- pg 478 #1,5,7,9,11,17,23,31,37
- Test 2.4, 6.1, 6.2, 6.5 on Friday 5/3

Objective:

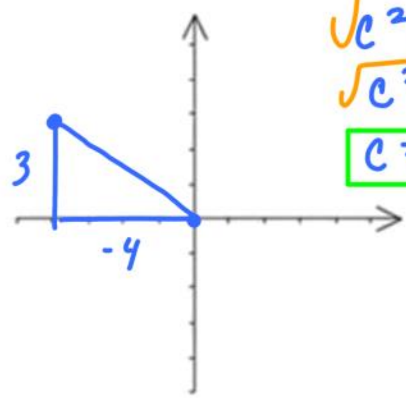
SWBAT: Plot complex numbers in complex plane and find the absolute value; convert between standard and trig form of a complex number

Do Now: Plot the point (-4, 3). Find its distance to the origin.

$$\sqrt{c^2} = \sqrt{(-4)^2 + (3)^2}$$

$$\sqrt{c^2} = \sqrt{25}$$

$$\boxed{c = 5}$$

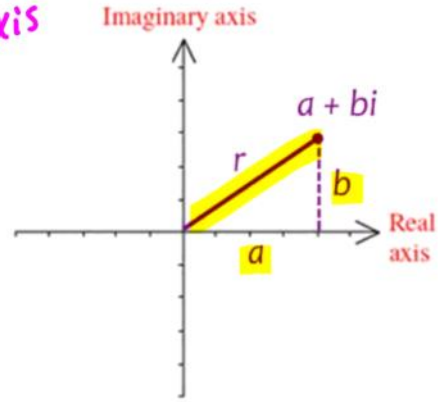


The Complex Plane

YAXIS

- The complex number $z = a + bi$ is represented by the coordinate (a, b)

XAXIS



- The ***absolute value*** of the complex number $z = a + bi$ is the point's distance from the origin.
- It is also known as **r** or the ***modulus*** **of z.**

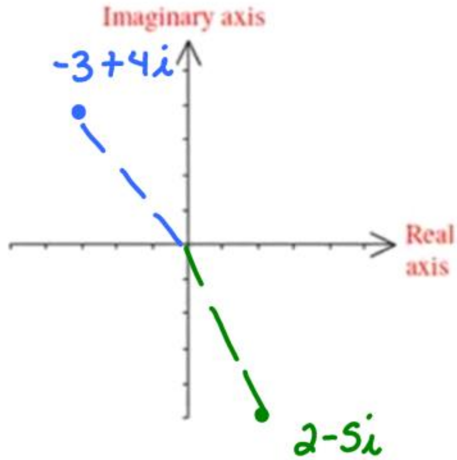
The Absolute Value of a Complex Number

$$|a+bi| = r = \sqrt{a^2+b^2}$$

Ex 1: Plot the complex numbers and find the absolute value.

$z_1 = -3 + 4i$ $z_2 = 2 - 5i$

$a = -3$	$a = 2$
$b = 4$	$b = -5$
$r = \sqrt{(-3)^2 + (4)^2}$	$r = \sqrt{(2)^2 + (5)^2}$
$r = \sqrt{25}$	$\boxed{r = \sqrt{29}}$
$\boxed{r = 5}$	



You try...

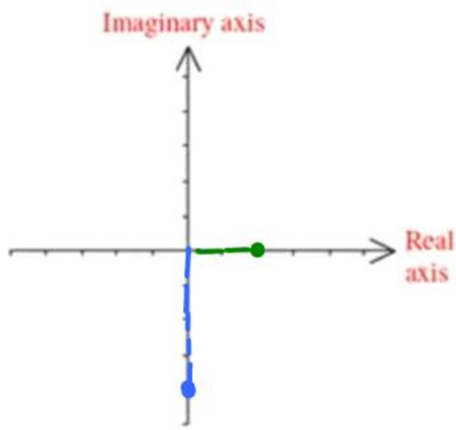
$$|a+bi| = r = \sqrt{a^2+b^2}$$

Ex 2: Plot the complex numbers and find the absolute value.

$z_1 = -4i$ $z_2 = 2$

$a=0$
$b=-4$
$r=\sqrt{a^2+b^2}$
$=\sqrt{0^2+(-4)^2}$
$=\sqrt{16}$
$=\boxed{4}$

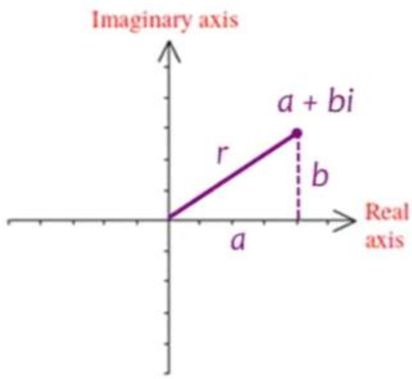
$a=2$
$b=0$
$r=\sqrt{a^2+b^2}$
$=\sqrt{(2)^2+0^2}$
$=\sqrt{4}$
$=\boxed{2}$



Trig Form of a Complex Number

Standard Form: $z = a + bi$

Trig Form: $z = r(\cos\theta + i\sin\theta)$



Where $a = r\cos\theta$

$b = r\sin\theta$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a}$

$0 \le \theta \le 2\pi$

Since there are an infinite number of coterminal angles for θ, we generally restrict θ to $0 \le \theta < 2\pi$.

Trig Form of a Complex Number

Where $a = r\cos\theta$
$b = r\sin\theta$
$r = \sqrt{a^2 + b^2}$
$\tan\theta = \frac{b}{a}$
$0 \le \theta \le 2\pi$

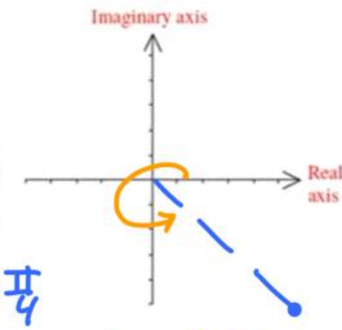
Standard Form: $z = a + bi$
Trig Form: $z = r(\cos\theta + i\sin\theta)$

Ex 3: Write the complex number in trig form:

$$z = 6 - 6i$$ $a=6 \quad b=-6$

1. Find r $= \sqrt{a^2+b^2} = \sqrt{(6)^2+(-6)^2}$
   $= \sqrt{72} = 6\sqrt{2}$

2. Find θ $\tan\theta' = \frac{b}{a}$ $\tan\theta' = \frac{-6}{6}$
   (Identify quadrant & use $\tan\theta = b/a$)
   $\tan\theta' = -1$
   $\theta' = 45° = \frac{\pi}{4}$



3. Rewrite in trig form. $\theta = \frac{7\pi}{4}$ Quadrant IV

$z = r(\cos\theta + i\sin\theta)$

$$\boxed{z = 6\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$$

You try...

Standard Form: $z = a + bi$

Trig Form: $z = r(\cos\theta + i\sin\theta)$

Where $a = r\cos\theta$

$b = r\sin\theta$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a}$

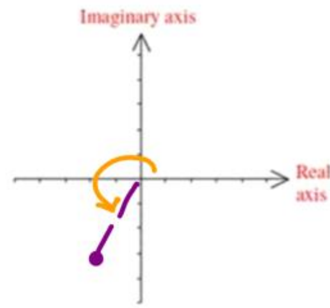
$0 \le \theta \le 2\pi$

Ex 4: Write the complex number in trig form:

$$z = -2 - 2\sqrt{3}i$$

$a = -2 \quad b = -2\sqrt{3}$

≈ -3.5



$r = \sqrt{a^2+b^2}$

$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2}$

$r = \sqrt{4+12} = \sqrt{16}$

$r = 4$

$\tan\theta' = \dfrac{b}{a}$

$\tan\theta' = \dfrac{-2\sqrt{3}}{-2}$

$\tan\theta' = \sqrt{3}$

$\theta' = 60° = \dfrac{\pi}{3}$

* in Quadrant III

$\theta = \dfrac{4\pi}{3}$

$z = r(\cos\theta + i\sin\theta)$

$$\boxed{z = 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)}$$

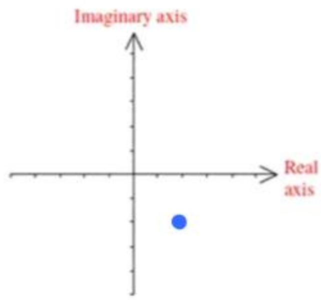
Converting between Trig and Standard forms

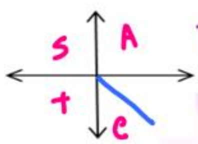
Standard Form: $z = a + bi$

Trig Form: $z = r(\cos\theta + i\sin\theta)$

Ex 4: Represent the complex number graphically and find the standard form of the number.

$$z=\sqrt{8}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$$





1. Evaluate $\cos\theta$ and $\sin\theta$

$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\sin\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\theta' = \frac{\pi}{4}$

2. Distribute r and simplify

$$z = \sqrt{8}\left(\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= \frac{\sqrt{16}}{2} - i\,\frac{\sqrt{16}}{2}$$

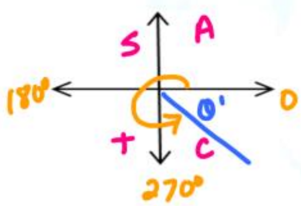
$$= \boxed{2 - 2i}$$

You try...

Standard Form: $z = a + bi$

Trig Form: $z = r(\cos\theta + i\sin\theta)$

Ex 5: Represent the complex number graphically and find the standard form of the number.



$\theta' = 60°$

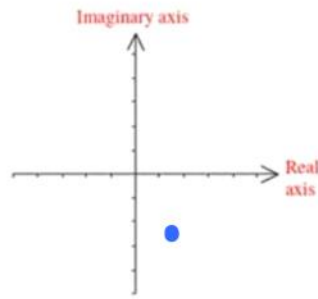
$$z = 3(\cos 300° + i\sin 300°)$$

$\cos 300° = \frac{1}{2}$ $\quad \sin 300° = -\frac{\sqrt{3}}{2}$

$z = 3\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$$\boxed{z = \frac{3}{2} + -\frac{3\sqrt{3}}{2}i}$$

≈ -2.6



Closure...

Standard Form: $z = a + bi$

Trig Form: $z = r(\cos\theta + i\sin\theta)$

Where $a = r\cos\theta$

$b = r\sin\theta$

$r = \sqrt{a^2 + b^2}$ $\qquad 0 \le \theta \le 2\pi$

$\tan\theta = \frac{b}{a}$

Represent the complex number graphically, and find the trig form of the complex number.

$$z = 3 - i$$

$a = 3 \quad b = -1$

$r = \sqrt{a^2 + b^2}$

$r = \sqrt{(3)^2 + (-1)^2}$

$r = \sqrt{10}$

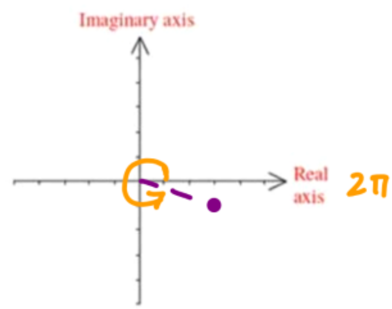
$\tan\theta' = \frac{b}{a}$

$\tan\theta' = -\frac{1}{3}$

$\theta' \approx -.3218$

$\theta = 2\pi - .3218$

$\theta \approx 5.96$



$z = r(\cos\theta + i\sin\theta)$

$= \boxed{\sqrt{10}(\cos 5.96 + i\, 5.96)}$

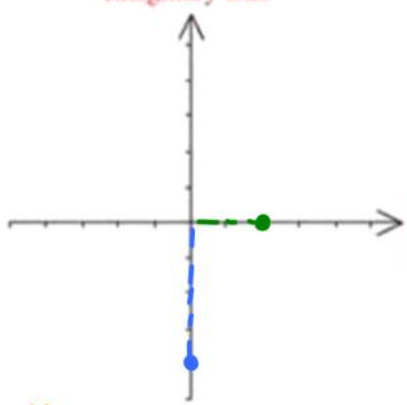
If time permits...

Represent the complex number graphically, and find the trig form of the complex number.

$z_1 = -4i \quad z_2 = 2$

$a=0 \qquad a=2$

$b=-4 \qquad b=0$



$r = \sqrt{a^2+b^2}$

$r = \sqrt{0^2+(-4)^2} \qquad r = \sqrt{2^2+0^2}$

$r = 4 \qquad r = 2$

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{-4}{0} \qquad \tan\theta = \frac{0}{2}$

$\tan\theta = \text{undef.} \qquad \tan\theta = 0 \quad \frac{y}{x}$

$\theta = \frac{3\pi}{2} \qquad \theta = 0$

$z = r(\cos\theta + i\sin\theta)$

$\boxed{z = 4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)} \qquad \boxed{z = 2(\cos 0 + i\sin 0)}$