Thursday, May 03, 2018
6:11 PM

Name: KEY Date: _____ Period: _____

Precalculus Section 6.5 Review

Formulas to know:

$z = a + bi$ (standard form of complex number)

$z = r(\cos\theta + i\sin\theta)$ (trigonometric form of complex number)

$r = \sqrt{a^2 + b^2}$ $\tan\theta = \frac{b}{a}$

$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$

$z^n = r^n(\cos n\theta + i\sin n\theta)$

Objective: To review operations with complex numbers in trigonometric form.

DO NOW:

1. Write in trigonometric form:

a. $z_1 = 5i$ $a=0$ $b=5$ $r=5$ $\tan\theta' = \frac{b}{a} = \frac{5}{0}$ = undef. $\theta = \frac{\pi}{2}$

$z = 5(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

b. $z_2 = -3\sqrt{3} - 3i$ $a=-3\sqrt{3}$ $b=-3$ $r = \sqrt{(-3\sqrt{3})^2 + (-3)^2} = \sqrt{36} = 6$ $\tan\theta' = \frac{b}{a} = \frac{-3}{-3\sqrt{3}} = \frac{1}{\sqrt{3}}$ $\theta' = \frac{\pi}{6}$ $\theta = \frac{7\pi}{6}$

$z = 6(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$

c. $z_3 = 4 + 7i$ $a=4$ $b=7$ $r = \sqrt{4^2 + 7^2} = \sqrt{65}$ $\tan\theta' = \frac{b}{a} = \frac{7}{4}$ $\theta' = 1.05$ QI $\theta \approx 1.05$

$z = \sqrt{65}(\cos 1.05 + i\sin 1.05)$

2. Find $(z_2)^8$. Write your answer in trigonometric form.

$z^8 = r^n(\cos n\theta + i\sin n\theta)$ $r=6$ $n=8$

$= 6^8[\cos(8 \cdot \frac{2\pi}{3}) + i\sin(8 \cdot \frac{2\pi}{3})]$

$= 6^8(\cos\frac{28\pi}{3} + i\sin\frac{28\pi}{3})$

$= 1,679,616(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$

* Subtract mults. of 2π or $\frac{6\pi}{3}$:

Class practice problems:

1. Graph the complex number and find the trigonometric form: $z = 1 + 3i$

$a=1$ $b=3$ $r = \sqrt{a^2 + b^2}$ $r = \sqrt{(1)^2 + (3)^2}$ $r = \sqrt{10}$ $\tan\theta' = \frac{b}{a}$ $\tan\theta' = \frac{3}{1} = 3$ $\theta' \approx 1.249$ QI $\theta \approx 1.249$

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{10}(\cos 1.249 + i\sin 1.249)$

2. Graph the complex number and find the standard form: $z = 3(\cos 150° + i\sin 150°)$

$\theta' = 30$

$z = 3(-\frac{\sqrt{3}}{2} + i(\frac{1}{2}))$ ← evaluate

$z = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$a = -\frac{3\sqrt{3}}{2} \approx -2.6$ $b = \frac{1}{2}$

3. Write in trig form, perform the indicated operation, then check in standard form: $(\sqrt{3}+i)(1+i)$

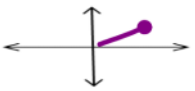
z_1 $a=\sqrt{3}$, $b=1$

$r=\sqrt{a^2+b^2}$

$r=\sqrt{(\sqrt{3})^2+1^2}$

$r=\sqrt{4}$

$r=2$



$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{1}{\sqrt{3}}$

$\theta' = \frac{\pi}{6}$

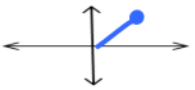
$\theta = \frac{\pi}{6}$

z_2 $a=1$, $b=1$

$r=\sqrt{a^2+b^2}$

$r=\sqrt{(1)^2+(1)^2}$

$r=\sqrt{2}$



$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{1}{1} = 1$

$\theta' = \frac{\pi}{4}$

$\theta = \frac{\pi}{4}$

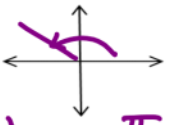
TRIG FORM: $z = r(\cos\theta + i\sin\theta)$

$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z_1 z_2 = r_1 r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

$= 2\sqrt{2}\left[\cos\left(\frac{\pi}{6}\cdot\frac{3}{3} + \frac{\pi}{4}\cdot\frac{3}{3}\right) + i\sin\left(\frac{\pi}{6}\cdot\frac{3}{3} + \frac{\pi}{4}\cdot\frac{3}{3}\right)\right]$



$\frac{\pi}{12}$

$= 2\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

$\approx .732 + 2.732i$ * Check at bottom of pg

4. Use DeMoivre's theorem to find $(2+2i)^6$

$r=\sqrt{2^2+2^2}$

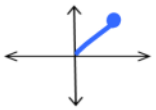
$r=\sqrt{8}=2\sqrt{2}$

$\tan\theta' = \frac{b}{a}$

$\tan\theta' = \frac{2}{2} = 1$

$\theta' = \frac{\pi}{4}$

in QI $\theta = \frac{\pi}{4}$



TRIG FORM: $z = r(\cos\theta + i\sin\theta)$

$z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

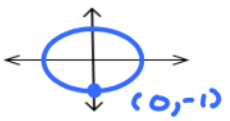
$z^n = r^n(\cos n\theta + i\sin n\theta)$ $r=2\sqrt{2}$ $n=6$

$z^6 = (2\sqrt{2})^6\left[\cos\left(6\cdot\frac{\pi}{4}\right) + i\sin\left(6\cdot\frac{\pi}{4}\right)\right]$

$= 512\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$= 512(0 + i(-1))$

$= -512i$



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check!

3) $(\sqrt{3}+i)(1+i) = \sqrt{3} + \sqrt{3}i + i + i^2$

$= \sqrt{3} + (\sqrt{3}+1)i + -1$

$= (\sqrt{3}-1) + (\sqrt{3}+1)i$

$\approx .732 + 2.732i$ ✓ answers match