Tuesday, March 26, 2019
7:39 PM

KEY

6.1B - Law of Sines & Ambiguous Case

Homework:
- Section 6.1B, #6,19-24 all
- Quarterly on 5.1-5.5, 6.1

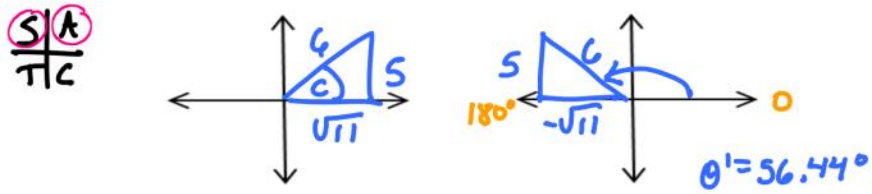
Objective:

SWBAT: Use Law of Sines to solve oblique SSA triangles.

Do Now:

Find all solutions in the interval [0, 360°):

$$\sin C = \frac{5}{6}$$



$\sin C = \frac{5}{6}$

$\sin^{-1}\left(\frac{5}{6}\right) = C$

$\boxed{C \approx 56.44°}$

$180° - 56.44° = \boxed{123.56°}$

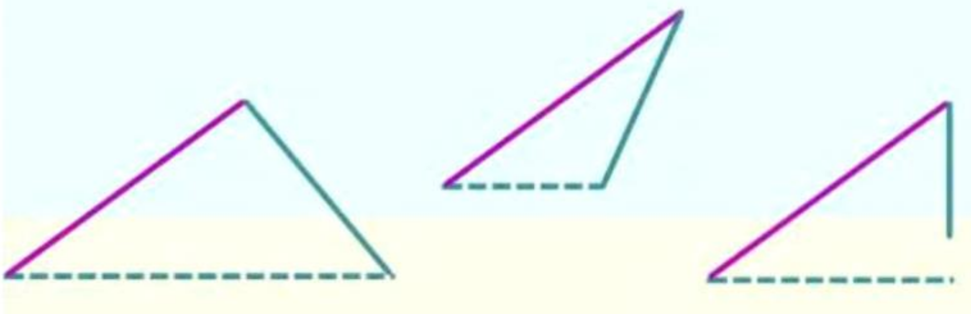
Homework questions??

Law of Sines & the Ambiguous Case

SSA triangle can have:

- no solution
- one solution
- two solutions

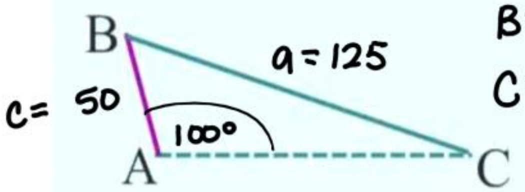
When presented with SSA, the best way to proceed is to draw the triangle with the given angle in the lower left corner and adjacent side going "up", as follows:

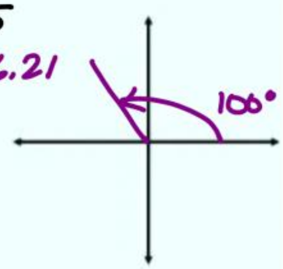


SSA given Obtuse Angle

Solve the triangle given A = 100°, a = 125 and c = 50.

A = 100°	a = 125
B = 56.8°	b = 106.21
C = 23.20°	c = 50





$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{125}{\sin 100°} = \frac{50}{\sin C}$$

$$\frac{125 \sin C}{125} = \frac{50 \sin 100°}{125}$$

$$\sin C = \frac{50 \sin 100°}{125} \leftarrow \approx .39$$

$$\sin^{-1}\left(\frac{50 \sin 100°}{125}\right) = C$$

$$\boxed{C \approx 23.20°}$$

$$B = 180° - A - C$$

$$B = 180° - 100° - 23.20°$$

$$\boxed{B \approx 56.8°}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{125}{\sin 100°} = \frac{b}{\sin 56.8°}$$

$$\frac{125 \sin 56.8°}{\sin 100°} = \frac{b \sin 100°}{\sin 100°}$$

$$\boxed{b \approx 106.21}$$

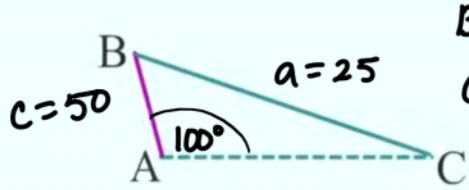
Is it possible to have two triangles?

* not with obtuse angle
* sum of Angles = 180° (obtuse angle is too big)

↙ 1 or no solutions

SSA given Obtuse Angle

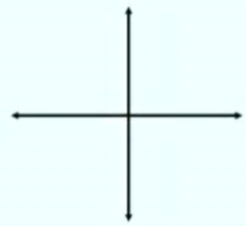
Solve the triangle given A = 100°, a = 25 and c = 50.



$A = 100°$ $a = 25$

$B =$ $b =$

$C =$ $c = 50$



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{25}{\sin 100°} = \frac{50}{\sin C}$$

$$\frac{50 \sin 100°}{25} = \frac{25 \sin C}{25}$$

$$\sin C = \frac{50 \sin 100°}{25} \approx 1.97$$

$\sin C$ can't $= 1.97$

$$\sin^{-1}\left(\frac{50 \sin 100°}{25}\right) = C$$

$\boxed{\text{no solution}}$

Will you always have a triangle?

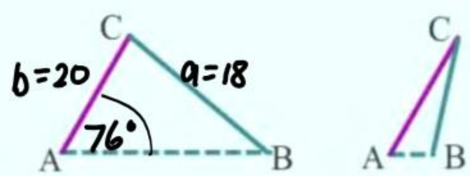
SSA given Acute Angle

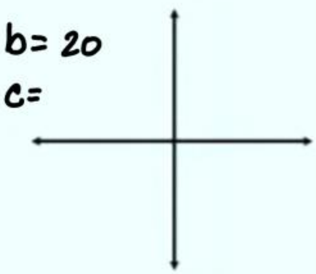
Solve the triangle given A = 76°, a = 18 and b = 20.

$A = 76°$ $\quad a = 18$

$B =$ $\quad b = 20$

$C =$ $\quad c =$





Start by setting up Law of Sines to find missing angle. Check to make sure value of sinB is in range of sine.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{18}{\sin 76°} = \frac{20}{\sin B}$$

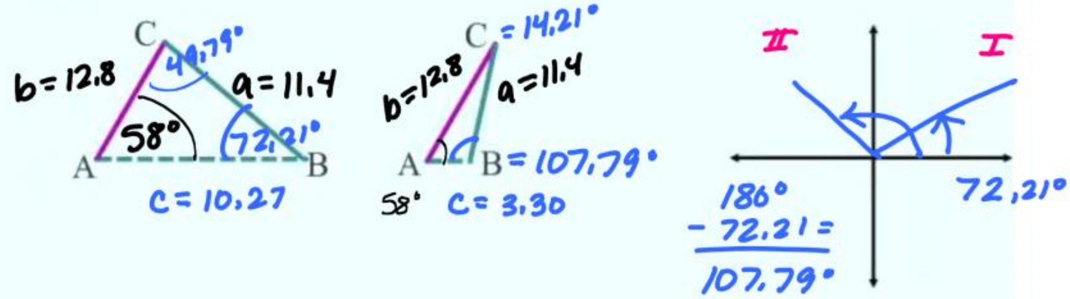
$$\frac{20 \sin 76°}{18} = \frac{18 \sin B}{18}$$

$$\sin B = \frac{20 \sin 76°}{18} \quad 1.07$$

$$\sin^{-1}\left(\frac{20 \sin 76°}{18}\right) = B \qquad \boxed{\text{no solution}}$$

SSA given Acute Angle

Solve the triangle given A = 58º, a = 11.4 and b = 12.8.



Start by setting up Law of Sines to find missing angle. There are two possible angles with that value of sine - an acute angle in QI and an obtuse angle in QII.

* 1ST TRIANGLE *Are both triangles possible??* ← yes!

$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\quad \frac{11.4}{\sin 58°} = \frac{12.8}{\sin B}$

$\frac{12.8 \sin 58°}{11.4} = \frac{\cancel{11.4} \sin B}{\cancel{11.4}}$

$\sin^{-1}\left(\frac{12.8 \sin 58°}{11.4}\right) = B$ ← ~.95

$\boxed{B \approx 72.21°}$

$C = 180° - 58° - 72.21°$

$\boxed{C = 49.79°}$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{11.4}{\sin 58°} = \frac{c}{\sin 49.79°}$

$\frac{c \sin 58°}{\sin 58°} = \frac{11.4 \sin 49.79°}{\sin 58°}$ $\quad \boxed{c \approx 10.27}$

* 2ND TRIangle:

$B = 180° - 72.21°$ $\quad \boxed{B = 107.79°}$

$C = 180° - A - B$

$C = 180° - 58° - 107.79°$ $\quad \boxed{C \approx 14.21°}$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{11.4}{\sin 58°} = \frac{c}{\sin 14.21°}$

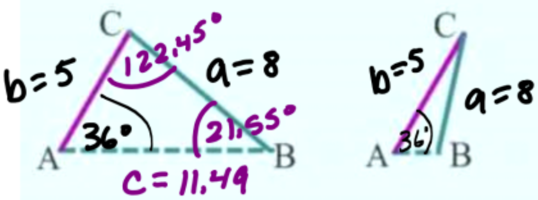
$\frac{c \sin 58°}{\sin 58°} = \frac{11.4 \sin 14.21°}{\sin 58°}$

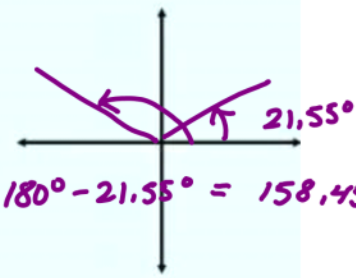
$\boxed{c \approx 3.30}$

* 0, 1, or 2 solutions

SSA given Acute Angle

Solve the triangle given A = 36°, a = 8 and b = 5.





$B = 180° - 21.55° = 158.45°$

How many triangles can we draw?

* 1st Triangle

$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\quad \frac{8}{\sin 36°} = \frac{5}{\sin B}$

$\frac{8 \sin B}{8} = \frac{5 \sin 36°}{8}$

$\sin B = \frac{5 \sin 36°}{8}$

$\sin^{-1}\left(\frac{5 \sin 36°}{8}\right) = B$

$\boxed{B \approx 21.55°}$

$C = 180° - A - B$

$C = 180° - 36° - 21.55°$

$\boxed{C \approx 122.45°}$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{8}{\sin 36°} = \frac{c}{\sin 122.45°}$

$\frac{c \sin 36°}{\sin 36°} = \frac{8 \sin 122.45°}{\sin 36°}$

$\boxed{c \approx 11.49}$

* 2nd Triangle:

$B = 180° - 21.55° = 158.45°$

$C = 180° - A - B$

$C = 180° - 36° - 158.45°$

$C = \cancel{-14.45°}$ not possible

* only 1 solution

Law of Sines & SSA summary...

1. Given **obtuse angle** - One or no solution.
   - *Do sides lengths make sense?*
   - *Is the sine of the angle in the range of the sine function?*

2. Given **acute angle** - One, two or no solutions.
   - *Use Law of Sines to set-up proportion to find missing angle.*
   - *If sine > 1, no solution!*
   - *If sine < 1, find angles in QI and QII.*
   - *Are both triangles possible?*
     YES, 2 solutions. NO, 1 solution.

You try...

Determine the number of triangles that can be formed and solve the triangles:

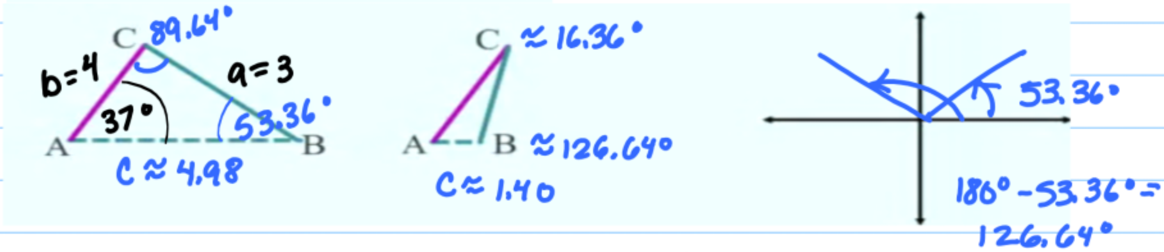
1. $A = 37°$, $a = 3$ and $b = 4$

2. $A = 30°$, $a = 50$ and $b = 100$

3. $A = 110°$, $a = 125$ and $b = 200$

*Acute: 0, 1 or 2 Solutions

1. $A = 37°$, $a = 3$ and $b = 4$



1st Δ:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{3}{\sin 37°} = \frac{4}{\sin B}$$

$$\frac{3\sin B}{3} = \frac{4\sin 37°}{3}$$

$$\sin^{-1}\left(\frac{4\sin 37°}{3}\right) = B$$

$$\boxed{B \approx 53.36°}$$

$C = 180° - A - B$

$C = 180° - 37° - 53.36°$ $\boxed{C \approx 89.64°}$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{3}{\sin 37°} = \frac{c}{\sin 89.64°}$$

$$\frac{3\sin 89.64°}{\sin 37°} = \frac{c\sin 37°}{\sin 37°}$$

$$\boxed{c \approx 4.98}$$

2nd Δ:

$B = 180° - 53.36°$ $\boxed{B \approx 126.64°}$

$C = 180° - A - B$

$C = 180° - 37° - 126.64°$ $\boxed{C \approx 16.36°}$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

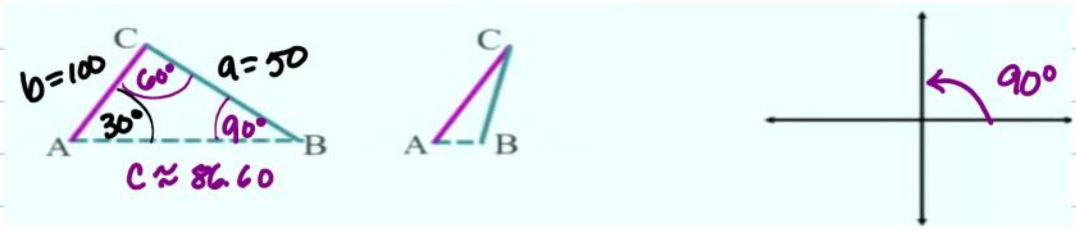
$$\frac{3}{\sin 37°} = \frac{c}{\sin 16.36°}$$

$$\frac{3\sin 16.36°}{\sin 37°} = \frac{c\sin 37°}{\sin 37°}$$

$$\boxed{c \approx 1.40}$$

SSA ← acute so 0, 1 or 2 solutions

2. $A = 30°$, $a = 50$ and $b = 100$



1st Δ:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{50}{\sin 30°} = \frac{100}{\sin B}$$

$$\frac{50 \sin B}{50} = \frac{100 \sin 30°}{50}$$

$$\sin^{-1}\left(\frac{100 \sin 30°}{50}\right) = B$$

$$\boxed{B = 90°}$$

$C = 180° - A - B$

$C = 180° - 30° - 90°$ $\boxed{C = 60°}$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{50}{\sin 30°} = \frac{c}{\sin 60°}$$

$$\frac{50 \sin 60°}{\sin 30°} = \frac{c \sin 30°}{\sin 30°}$$

$$\boxed{c \approx 86.60}$$

2nd Δ: * not possible