

Wednesday, March 14, 2018
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Section 5.5D: Half-Angle Formulas

Name: KEY

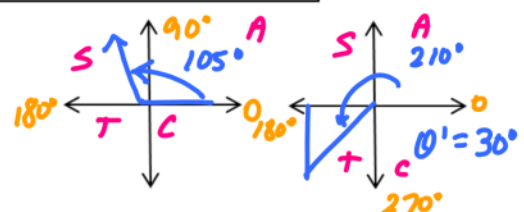
Fill in the following chart from memory!

* Sign depends on Quadrant of $\frac{u}{2}$

Half-Angle Formulas	
$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$	$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$
$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u}$	$\tan \frac{u}{2} = \frac{\sin u}{1 + \cos u}$

Example Problems:

1. Find cosine and tangent of 105° .

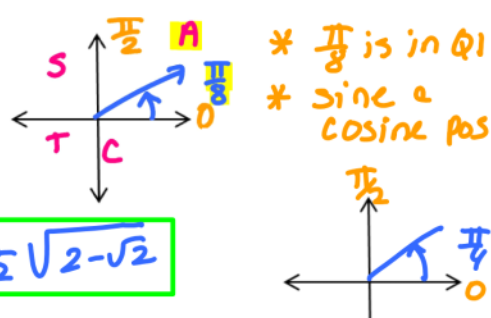
$$\cos 105^\circ = \cos\left(\frac{210^\circ}{2}\right) = -\sqrt{\frac{1 + \cos 210^\circ}{2}}$$


$$= -\sqrt{\frac{1 + (-\frac{\sqrt{3}}{2})}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2} \cdot \frac{1}{2}} = \boxed{-\frac{1}{2}\sqrt{2 - \sqrt{3}}}$$

$$\tan 105^\circ = \tan\left(\frac{210^\circ}{2}\right) = \frac{1 - \cos 210^\circ}{\sin 210^\circ} = \frac{1 - (-\frac{\sqrt{3}}{2})}{-\frac{1}{2}} = \frac{2 + \sqrt{3}}{-1} = \boxed{-2 - \sqrt{3}}$$

2. Find the exact value of $\sin \frac{\pi}{8}$, $\cos \frac{\pi}{8}$ and $\tan \frac{\pi}{8}$.

* MULT by 2 to get u $\frac{\pi}{8} \cdot 2 = \frac{\pi}{4}$

$$\sin \frac{\pi}{8} = \sin\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1 - \cos \frac{\pi}{4}}{2}}$$


$$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{2} \cdot \frac{1}{2}} = \boxed{\frac{1}{2}\sqrt{2 - \sqrt{2}}}$$

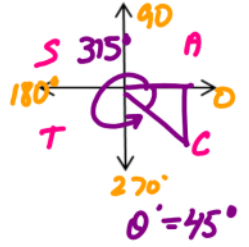
$$\cos \frac{\pi}{8} = \cos\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2} \cdot \frac{1}{2}} = \boxed{\frac{1}{2}\sqrt{2 + \sqrt{2}}}$$

$$\tan \frac{\pi}{8} = \tan\left(\frac{\frac{\pi}{4}}{2}\right) = \frac{1 - \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \frac{(2 - \sqrt{2}) \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{2} - 2}{2} = \boxed{\sqrt{2} - 1}$$

3. Find the exact value of $\tan 157^\circ 30'$.

$$\tan 157^\circ 30' = \tan\left(\frac{315^\circ}{2}\right) = \frac{1 - \cos 315^\circ}{\sin 315^\circ} = \frac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$$

$$= \frac{2 - \sqrt{2}}{2} \cdot \left(-\frac{2}{\sqrt{2}}\right) = \frac{(2 - \sqrt{2}) \cdot \sqrt{2}}{-\sqrt{2}} = \frac{2\sqrt{2} - 2}{-2} = \boxed{-\sqrt{2} + 1}$$



4. Find the exact values of $\sin \frac{u}{2}$, $\cos \frac{u}{2}$ and $\tan \frac{u}{2}$ using the half-angle formula, given:

$$\frac{b}{0} \csc u = -\frac{5}{3}; \pi < u < \frac{3\pi}{2}$$

$$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$$

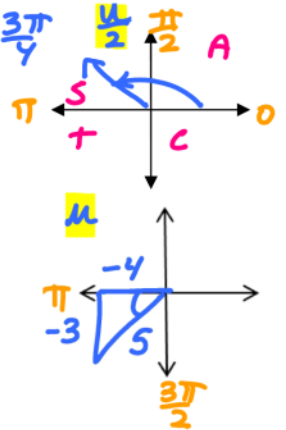
$$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$$

$$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$$

$$\begin{aligned} \sin \frac{u}{2} &= + \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \frac{-4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{5} \cdot \frac{1}{2}} \\ &= \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}} \end{aligned}$$

$$\begin{aligned} \cos \frac{u}{2} &= - \sqrt{\frac{1 + \cos u}{2}} = - \sqrt{\frac{1 + \frac{-4}{5}}{2}} = - \sqrt{\frac{\frac{1}{5}}{2}} = - \sqrt{\frac{1}{5} \cdot \frac{1}{2}} \\ &= -\frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{-\frac{\sqrt{10}}{10}} \end{aligned}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{-4}{5}}{-\frac{3}{5}} = \frac{\frac{9}{5}}{-\frac{3}{5}} = \frac{9}{-3} = \boxed{-3}$$



5. Find the exact values of $\sin \frac{u}{2}$, $\cos \frac{u}{2}$ and $\tan \frac{u}{2}$ using the half-angle formula, given:

$$\cos u = \frac{2}{7}; \frac{3\pi}{2} < u < 2\pi$$

$$\frac{1}{2} \frac{3\pi}{2} < \frac{u}{2} < \frac{2\pi}{2}$$

$$\frac{3\pi}{4} < \frac{u}{2} < \pi$$

$$\frac{3\pi}{4} < \frac{u}{2} < \pi$$

$$\begin{aligned} \sin \frac{u}{2} &= + \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \frac{2}{7}}{2}} = \sqrt{\frac{\frac{5}{7}}{2}} = \sqrt{\frac{5}{7} \cdot \frac{1}{2}} \\ &= \frac{\sqrt{5}}{\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \boxed{\frac{\sqrt{70}}{14}} \end{aligned}$$

$$\begin{aligned} \cos \frac{u}{2} &= - \sqrt{\frac{1 + \cos u}{2}} = - \sqrt{\frac{1 + \frac{2}{7}}{2}} = - \sqrt{\frac{\frac{9}{7}}{2}} = - \sqrt{\frac{9}{7} \cdot \frac{1}{2}} = -\frac{3}{\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \boxed{-\frac{3\sqrt{14}}{14}} \end{aligned}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{2}{7}}{-\frac{3\sqrt{5}}{7}} = \frac{\frac{5}{7}}{-\frac{3\sqrt{5}}{7}} = \frac{5}{-3\sqrt{5}} = \frac{5}{-3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{-3(5)} = \boxed{-\frac{\sqrt{5}}{3}}$$

