Saturday, March 09, 2019
9:53 AM

5.5 C – Half-Angle Formulas

KEY

Homework:
- Section 5.5 C, FACTORING WS 105 - 151 circled PROBLEMS — due FRIDAY 3/15
- 5.5 Mini Quiz – Tuesday 3/12
- 5.1 - 5.3 Re-Assessment – Thursday 3/14
- Ch. 5 Test
  - Monday 3/18 (Formulas)
  - Tues 3/19 or Weds 3/20 (Block Schedule)

Objective:
- Use half-angle and double angle formulas to rewrite & evaluate trig functions

Do Now:

Find all solutions of $\sin x - \frac{\sqrt{2}}{2}\sin 2x = 0$ in the interval $[0, 2\pi)$.

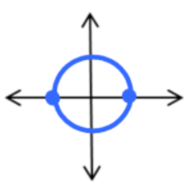
$\sin x - \frac{\sqrt{2}}{\cancel{2}}(\cancel{2}\sin x \cos x) = 0$

$\sin x - \sqrt{2}\sin x \cos x = 0$

$\sin x(1 - \sqrt{2}\cos x) = 0$

S | A
T | C

$\sin x = 0$

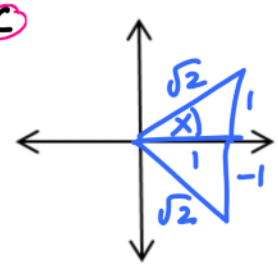


$\boxed{x = 0, \pi}$

$1 - \sqrt{2}\cos x = 0$

$-\sqrt{2}\cos x = -1$

$\cos x = \frac{1}{\sqrt{2}}$



$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$

Homework Questions?

Half-Angle Formulas

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

✰ *Use "All Students Take Calculus" to help identify the quadrant where u/2 is positive or negative.*

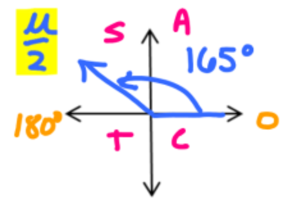
How do we determine the sign of cosine and sine functions for half-angles?

Examples

Half-Angle Formulas

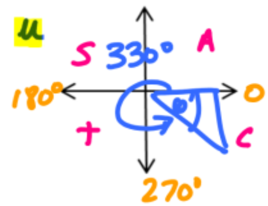
$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

Find the sine and cosine of 165°.



$$\sin 165° = \sin\left(\frac{330}{2}\right) = +\sqrt{\frac{1-\cos u}{2}}$$

sine is positive in QII, where 165° ($\frac{u}{2}$) lies.



$\theta' = 30°$

$$= \sqrt{\frac{1-\cos 330}{2}} = \sqrt{\frac{1-\left(\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}} \text{ or } \boxed{\frac{1}{2}\sqrt{2-\sqrt{3}}}$$

$$\cos 165° = \cos\left(\frac{330}{2}\right) = -\sqrt{\frac{1+\cos u}{2}}$$

cos is neg in QII, where 165° ($\frac{u}{2}$) lies.

$$= -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}}$$

$$= -\sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}} \text{ or } \boxed{-\frac{1}{2}\sqrt{2+\sqrt{3}}}$$

Examples

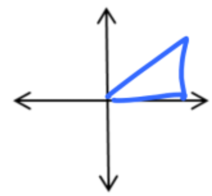
Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

Find the exact value of $\tan\frac{\pi}{12}$.

$\tan\frac{u}{2}$

$$\tan\frac{\pi}{12}^{*} = \tan\frac{\frac{2\pi}{12}}{2} = \tan\frac{\frac{\pi}{6}}{2} = \frac{1-\cos u}{\sin u}$$



* MULT BY 2 TO get u

$$= \frac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{2-\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2-\sqrt{3}}{2}\cdot\frac{2}{1} = \boxed{2-\sqrt{3}}$$

Examples

Half-Angle Formulas

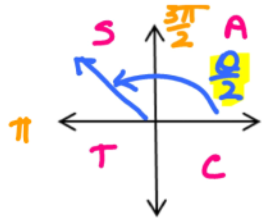
$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

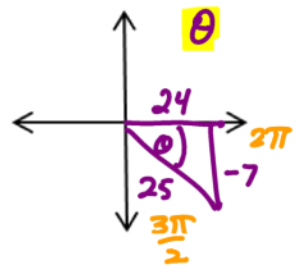
Find the $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$ and $\tan\frac{\theta}{2}$ given $\csc\theta = -\frac{25}{7}$, $\frac{3\pi}{2} < \theta < 2\pi$

$\frac{h}{o}$

$\frac{1}{2}\cdot\frac{3\pi}{2} < \frac{\theta}{2} < \frac{2\pi}{2}$ $\qquad$ $\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

* $\frac{\theta}{2}$ lies in Q II, where sine is pos





$$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{24}{25}}{2}} = \sqrt{\frac{\frac{1}{25}}{2}} = \sqrt{\frac{1}{25}\cdot\frac{1}{2}} =$$

$$= \frac{1}{5}\cdot\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{10}}$$

$$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{24}{25}}{2}} = -\sqrt{\frac{\frac{49}{25}}{2}} =$$

$$= -\sqrt{\frac{49}{25}\cdot\frac{1}{2}} = -\frac{7}{5}\sqrt{\frac{1}{2}} = -\frac{7}{5}\cdot\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-7\sqrt{2}}{10}}$$

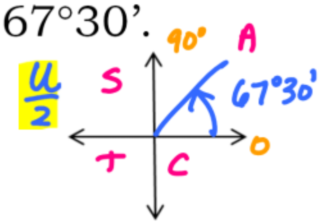
$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{24}{25}}{\frac{-7}{25}} = \frac{\frac{1}{25}}{\frac{-7}{25}} = \frac{1}{25}\cdot\frac{-25}{7} = \boxed{-\frac{1}{7}}$$

You Try…….

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

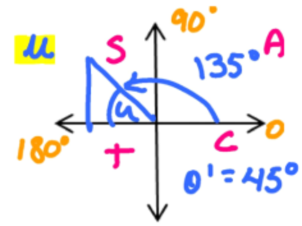
Find exact value of the sine and tangent of 67°30'.



$$\sin 67°30' = \sin\left(\frac{135°}{2}\right) = +\sqrt{\frac{1-\cos u}{2}}$$

* MULT BY 2 TO get u

* Pos. because 67°30' is in QI



$$= \sqrt{\frac{1-\cos 135°}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$= \boxed{\frac{1}{2}\sqrt{2+\sqrt{2}}}$$

$$\tan 67°30' = \tan\left(\frac{135°}{2}\right) = \frac{1-\cos 135°}{\sin 135°} = \frac{1-\frac{-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= \frac{\frac{2+\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2+\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = \frac{(2+\sqrt{2})}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{2}+2}{2} = \boxed{\sqrt{2}+1}$$

You Try.......

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

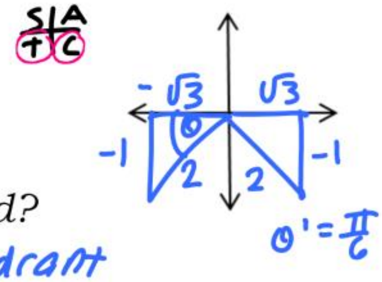
Use the half-angle formulas to simplify the expression:

$$+\sqrt{\frac{1+\cos 4x}{2}} = \left|\cos\frac{4x}{2}\right| = \left|\underline{\cos 2x}\right|$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \sin 2u = 2\sin u\cos u$$

Closure...

If I'm told that $\sin\,\theta = -\frac{1}{2}$, can I evaluate $\cos\,\frac{\theta}{2}$ and $\cos\,2\theta$?



Why or why not? NO

If not, what additional information is needed?

You need to know which quadrant θ is in.

If time permits.......

Section 5.5C - #35-38, 41, 43, 49, 50, 55