Tuesday, March 13, 2018
6:03 PM

Double-Angle Formulas

$\sin 2u = 2\sin u \cos u$

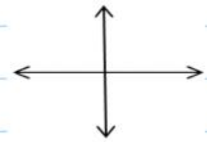
$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$

$\cos 2u = \cos^2 u - \sin^2 u$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

Find the exact solutions of the equation in the interval [0,2pi).



10) $\sin 2x + \cos x = 0$

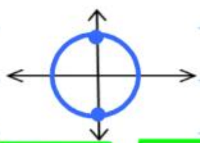
$2\sin x \cos x + \cos x = 0$

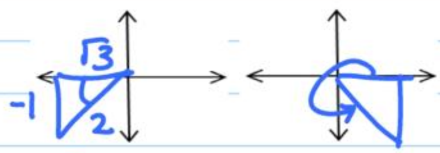
$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$





S | A
T | C

$\theta' = \frac{\pi}{6}$

$\boxed{x = \frac{\pi}{2}}$ $\boxed{x = \frac{3\pi}{2}}$ $\boxed{x = \frac{7\pi}{6}}$ $\boxed{x = \frac{11\pi}{6}}$

11) $4\sin x \cos x = 1$

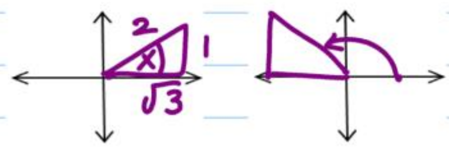
$2\sin x\cos x + 2\sin x \cos x = 1$

$\sin 2x + \sin 2x = 1$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

S | A
T | C



$\frac{1}{2}(2x) = (\frac{\pi}{6} + 2\pi n)\frac{1}{2}$ $\qquad$ $\frac{1}{2}(2x) = (\frac{5\pi}{6} + 2\pi n)\frac{1}{2}$

$x = \frac{\pi}{12} + \pi n$ $\qquad$ $x = \frac{5\pi}{12} + \pi n$

Solutions in the interval $[0, 2\pi)$:
$\frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}$

* ADD $\frac{12\pi}{12}$ until you get to 2π

12) $\sin 2x \sin x = \cos x$ $\qquad$ *Doubl - Angle formula

$2\sin x \cos x \sin x = \cos x$

$2\sin^2 x \cos x = \cos x$

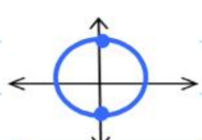
$2\sin^2 x \cos x - \cos x = 0$

$\cos x(2\sin^2 x - 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin^2 x = 1$

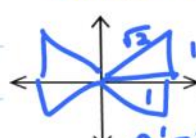
$\sin^2 x = \frac{1}{2}$ $\quad$ $\sin x = \pm\frac{1}{\sqrt{2}}$

$\cos x = 0$



$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}}$



$\theta' = \frac{\pi}{4}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

Double-Angle Formulas

$$\sin 2u = 2\sin u\cos u \qquad \cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} \qquad = 1 - 2\sin^2 u$$

14) $\cos 2x + \sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

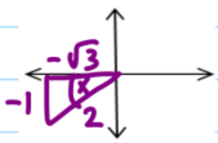
$-2\sin^2 x + \sin x + 1 = 0$

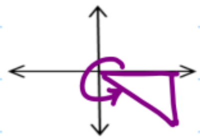
$-(2\sin^2 x - \sin x - 1) = 0 \qquad 2x^2 - x - 1$

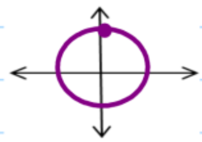
$-(2\sin x + 1)(\sin x - 1) = 0 \qquad (2x+1)(x-1)$

$2\sin x + 1 = 0 \qquad \sin x - 1 = 0$

S A / T C $\qquad \sin x = -\frac{1}{2} \qquad \sin x = 1$





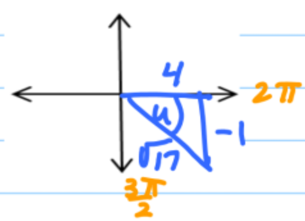


$\theta' = \frac{\pi}{6}$

$\boxed{x = \frac{7\pi}{6}} \qquad \boxed{x = \frac{11\pi}{6}} \qquad \boxed{x = \frac{\pi}{2}}$

Find the exact value of sin 2u, cos 2u and tan 2u using the double-angle formulas.

26) $\cot u = -4, \quad \frac{3\pi}{2} < u < 2\pi$



$\sin 2u = 2\sin u\cos u$

$= 2\left(-\frac{1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right) = \boxed{\frac{-8}{17}}$

$\cos 2u = \cos^2 u - \sin^2 u$

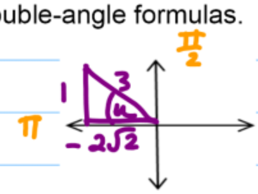
$\left(\frac{4}{\sqrt{17}}\right)^2 - \left(-\frac{1}{\sqrt{17}}\right)^2 = \frac{16}{17} - \frac{1}{17} = \boxed{\frac{15}{17}}$

$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(-\frac{1}{4}\right)}{1-\left(-\frac{1}{4}\right)^2} = \frac{-\frac{1}{2}}{1-\frac{1}{16}} = \frac{-\frac{1}{2}}{\frac{15}{16}}$

$= -\frac{1}{\cancel{2}_1} \cdot \frac{\cancel{16}^{8}}{15} = \boxed{\frac{-8}{15}}$

Find the exact value of sin 2u, cos 2u and tan 2u using the double-angle formulas.

$\frac{h}{o}$ 28) $\csc u = 3, \quad \frac{\pi}{2} < u < \pi$



$$\sin 2u = 2\sin u \cos u$$
$$= 2\left(\frac{1}{3}\right)\left(\frac{-2\sqrt{2}}{3}\right) = \boxed{\frac{-4\sqrt{2}}{9}}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= \left(-\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} - \frac{1}{9} = \boxed{\frac{7}{9}}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(\frac{1}{-2\sqrt{2}}\right)}{1-\left(\frac{1}{-2\sqrt{2}}\right)^2} = \frac{-\frac{1}{\sqrt{2}}}{1-\frac{1}{8}} = \frac{-\frac{1}{\sqrt{2}}}{\frac{7}{8}}$$

$$= -\frac{1}{\sqrt{2}} \cdot \frac{8}{7} = \frac{-8}{7\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-8\sqrt{2}}{14}$$

$$= \boxed{-\frac{4\sqrt{2}}{7}}$$