Thursday, March 07, 2019
6:53 PM

Section 5.5B: Double-Angle Formulas Name: KEY

Do Now: Fill in the following chart from memory!

Double-Angle Formulas

$\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$

Example Problems:

1. Solve $\cos 2x + \cos x = 0$.

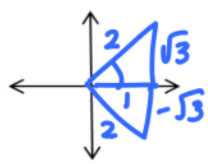
$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$ $2x^2 + x - 1$

$(2\cos x - 1)(\cos x + 1) = 0$ $(2x-1)(x+1)$

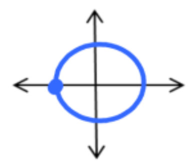
$\cos x = \frac{1}{2}$; $\cos x = -1$

$\cos x = \frac{1}{2}$



$\boxed{x = \frac{\pi}{3} + 2\pi n \quad x = \frac{5\pi}{3} + 2\pi n}$

$\cos x = -1$



$\boxed{x = \pi + 2\pi n}$

2. Find all solutions of $\sin 2x \sin x = 2\cos x$ in the interval $[0, 2\pi)$.

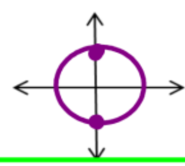
$2\sin x\cos x \sin x = 2\cos x$

$2\sin^2 x \cos x - 2\cos x = 0$

$2\cos x(\sin^2 x - 1) = 0$

$2\cos x = 0$

$\cos x = 0$

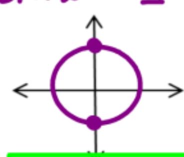


$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\sin^2 x - 1 = 0$

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$



$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

3. Find all solutions of $\cos 2\alpha + 5\sin\alpha + 2 = 0$ in the interval $[0, 2\pi)$.

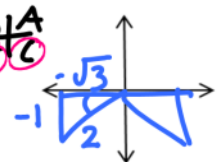
$1 - 2\sin^2\alpha + 5\sin\alpha + 2 = 0$

$-2\sin^2\alpha + 5\sin\alpha + 3 = 0$

$-(2\sin^2\alpha - 5\sin\alpha - 3) = 0$ $-(2x^2 - 5x - 3)$

$-(2\sin\alpha + 1)(\sin\alpha - 3) = 0$ $-(2x+1)(x-3)$

$\sin\alpha = -\frac{1}{2}$; $\sin\alpha = 3$ no solution



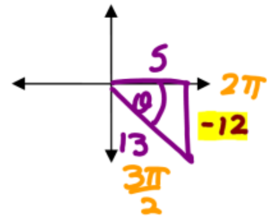
$\theta' = \frac{\pi}{6}$

$\boxed{\alpha = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\frac{h}{a}$

4. Find the exact value of the sin 2θ, cos 2θ, and tan 2θ given $\sec\theta = \frac{13}{5}$, $\frac{3\pi}{2} < \theta < 2\pi$.

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right) = \boxed{\frac{-120}{169}}$$



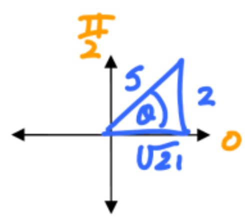
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(\frac{5}{13}\right)^2 - 1 = 2\left(\frac{25}{169}\right) - 1$$
$$= \frac{50}{169} - \frac{169}{169} = \boxed{\frac{-119}{169}}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{-\frac{24}{5}}{1-\frac{144}{25}} = \frac{-\frac{24}{5}}{-\frac{119}{25}} = \frac{-24}{\cancel{5}_1} \cdot \frac{\cancel{25}^{5}}{-119}$$
$$= \boxed{\frac{120}{119}}$$

$\frac{h}{o}$

5. Find the exact value of sin 2θ, cos 2θ and tan 2θ given $\csc\theta = \frac{5}{2}$, $0 < \theta < \frac{\pi}{2}$.

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right) = \boxed{\frac{4\sqrt{21}}{25}}$$



$$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{\sqrt{21}}{5}\right)^2 - 1 = 2\left(\frac{21}{25}\right) - 1$$
$$= \frac{42}{25} - \frac{25}{25} = \boxed{\frac{17}{25}}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{2}{\sqrt{21}}\right)}{1-\left(\frac{2}{\sqrt{21}}\right)^2} = \frac{\frac{4}{\sqrt{21}}}{1-\frac{4}{21}} = \frac{\frac{4}{\sqrt{21}}}{\frac{17}{21}} = \frac{4}{\sqrt{21}} \cdot \frac{21}{17}$$
$$= \frac{84}{17\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{84\sqrt{21}}{357} = \boxed{\frac{4\sqrt{21}}{17}}$$