Friday, March 09, 2018
11:35 AM

KEY

5.5 A – Multiple Angle Formulas

Homework: ♦ Section 5.5 A – Check your answers!

Objective:

- Use multiple angle formulas to rewrite & evaluate trig functions

Do Now: Handout

Find the following using sum formulas:

sin(2u) = sin(u + u) =

*cos(2u) = cos(u + u) =

**There are 3 ways you can write the equation for cos(2u)!*

tan(2u) = tan(u + u) =

Do Now

Find the following using sum formulas:

sin(2u) = sin(u + u) = $\sin x \cos x + \cos x \sin x$

$\sin x \cos x + \sin x \cos x$

$\boxed{2\sin x \cos x}$

*cos(2u) = cos(u + u) = $\cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= \boxed{2\cos^2 x - 1}$

**There are 3 ways you can write the equation for cos(2u)!*

$\boxed{\cos^2 x - \sin^2 x}$ $\quad 1 - \sin^2 x - \sin^2 x = \boxed{1 - 2\sin^2 x}$

tan(2u) = tan(u + u) = $\tan(x + x)$

$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

$$= \boxed{\frac{2\tan x}{1 - \tan^2 x}}$$

Examples

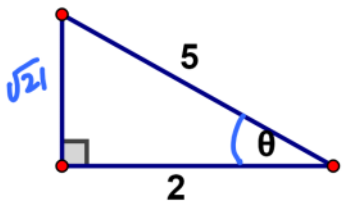
Double-Angle Formulas

$\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$

Use the figure to evaluate the exact value of the trig function.



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{21}}{5}\right)\left(\frac{2}{5}\right) = \boxed{\frac{4\sqrt{21}}{25}}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{2}{5}\right)^2 - 1$

$= 2\left(\frac{4}{25}\right) - 1$

$= \frac{8}{25} - \frac{25}{25} = \boxed{\frac{-17}{25}}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(\frac{\sqrt{21}}{2}\right)}{1-\left(\frac{\sqrt{21}}{2}\right)^2}$

$= \dfrac{\sqrt{21}}{1-\frac{21}{4}} = \dfrac{\sqrt{21}}{\frac{-17}{4}}$

$= \boxed{-\frac{4\sqrt{21}}{17}}$

Examples

Double-Angle Formulas

$$\sin 2u = 2\sin u \cos u \qquad \cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

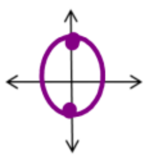
Find the exact solutions of 2cosx + sin2x = 0 in the interval [0, 2π).

$2\cos x + 2\sin x\cos x = 0$ * Double angle formula

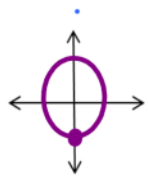
$2\cos x(1 + \sin x) = 0$ * FACTOR GCF!

$2\cos x = 0$ | $1 + \sin x = 0$

$\cos x = 0$ | $\sin x = -1$



$$\boxed{x = \frac{\pi}{2} \quad x = \frac{3\pi}{2}}$$



$$\boxed{x = \frac{3\pi}{2}}$$

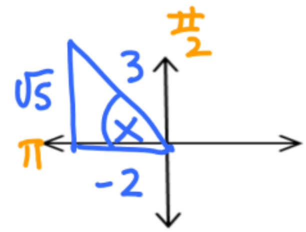
Examples

Double-Angle Formulas

$$\sin 2u = 2\sin u \cos u \qquad \cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

Find the exact values of sin(2x), cos(2x) and tan(2x) given

$$\cos x = -\frac{2}{3} \quad and \quad \frac{\pi}{2} < x < \pi$$



$$\sin 2x = 2\sin x\cos x = 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right) = \boxed{\frac{-4\sqrt{5}}{9}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(-\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2 = \frac{4}{9} - \frac{5}{9} = \boxed{\frac{-1}{9}}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(\frac{\sqrt{5}}{-2}\right)}{1-\left(\frac{\sqrt{5}}{-2}\right)^2} = \frac{-\sqrt{5}}{1-\frac{5}{4}} = \frac{-\sqrt{5}}{-\frac{1}{4}} = \boxed{4\sqrt{5}}$$

Your turn....

Double-Angle Formulas

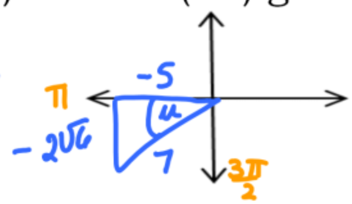
$$\sin 2u = 2\sin u \cos u \qquad \cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

Find the exact values of sin(2u), cos(2u) and tan(2u) given

$\frac{h}{a}$ $\sec u = -\frac{7}{5}$ *and* $\pi < u < \frac{3\pi}{2}$

$\sqrt{24} = 2\sqrt{6}$



$$\sin 2x = 2\sin x\cos x = 2\left(\frac{-2\sqrt{6}}{7}\right)\left(\frac{-5}{7}\right) = \boxed{\frac{20\sqrt{6}}{49}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(-\frac{5}{7}\right)^2 - \left(\frac{-2\sqrt{6}}{7}\right)^2 = \frac{25}{49} - \frac{24}{49} = \boxed{\frac{1}{49}}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(\frac{-2\sqrt{6}}{-5}\right)}{1-\left(\frac{-2\sqrt{6}}{5}\right)^2} = \frac{\frac{4\sqrt{6}}{5}}{1-\frac{24}{25}} = \frac{\frac{4\sqrt{6}}{5}}{\frac{1}{25}}$$

$$= \frac{4\sqrt{6}}{\cancel{5}_1} \cdot \frac{\cancel{25}^{5}}{1} = \boxed{20\sqrt{6}}$$

Closure ...

Double-Angle Formulas

$$\sin 2u = 2\sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

Find the exact solutions of cos2x + sinx = 0 in the interval [0, 2π).

$1 - 2\sin^2 x + \sin x = 0$

$-2\sin^2 x + \sin x + 1 = 0$

$-1(2\sin^2 x - \sin x - 1) = 0$

$-(2\sin x + 1)(\sin x - 1) = 0$

$2x^2 - x - 1$

$(2x+1)(x-1)$

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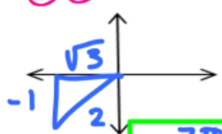
$2\sin x + 1 = 0$

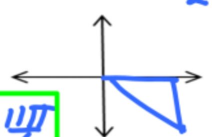
$\sin x = -\frac{1}{2}$

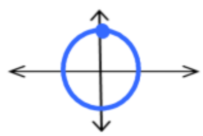
$\sin x = 1$



$\theta' = \frac{\pi}{6}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$





$\boxed{x = \frac{\pi}{2}}$

What form of the cosine double angle formula will be most useful in this example?