Thursday, February 28, 2019
7:07 PM

KEY

5.4 C – Sum and Difference Formulas

Homework:
- Section 5.4 C (VC)
- Quiz - Monday

Objectives: Use sum & difference formulas to:

- Evaluate trig functions
- Verify identities
- Solve trig equations

Do Now:

- Discuss HW solutions with partner

Homework Questions??

Finding Sum & Differences

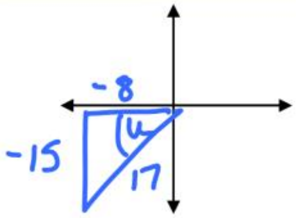
Given $\tan u = \dfrac{15}{8}$ and $\tan v = \dfrac{-3}{4}$.

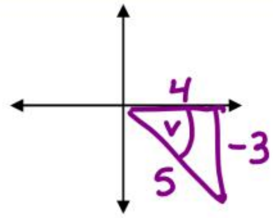
Angle ***u*** is in Quadrant III and Angle **v** is in Quadrant IV. Find the exact value of each.

$c^2 = 15^2 + 8^2$

$\sqrt{c^2} = \sqrt{289}$

$c = 17$





$sin(u-v) = \sin u \cos v - \cos u \sin v$

$\left(-\frac{15}{17}\right)\left(\frac{4}{5}\right) - \left(\frac{-8}{17}\right)\left(\frac{-3}{5}\right) = \frac{-60}{85} - \frac{24}{85} = \boxed{\frac{-84}{85}}$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{\frac{15}{8} + \frac{-3}{4}\cdot\frac{2}{2}}{1 - \left(\frac{15}{8}\right)\left(\frac{-3}{4}\right)} = \dfrac{\frac{9}{8}}{\frac{77}{32}}$

$\frac{9}{\cancel{8}_{1}} \cdot \frac{\cancel{32}^{4}}{77} = \boxed{\frac{36}{77}}$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

$= \left(-\frac{8}{17}\right)\left(\frac{4}{5}\right) + \left(-\frac{15}{17}\right)\left(\frac{-3}{5}\right)$

$= -\frac{32}{85} + \frac{45}{85}$

$= \boxed{\frac{13}{85}}$

Using sum & differences to verify identities

$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$$

$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x \sin^2 y$

$= 1 - \sin^2 y - \sin^2 x + \cancel{\sin^2 x \sin^2 y} - \cancel{\sin^2 x \sin^2 y}$

$= 1 - \sin^2 x - \sin^2 y$

$= \cos^2 x - \sin^2 y$ ✓

Using sum and differences to find solutions

Find all the solutions in the interval [0, 2π):

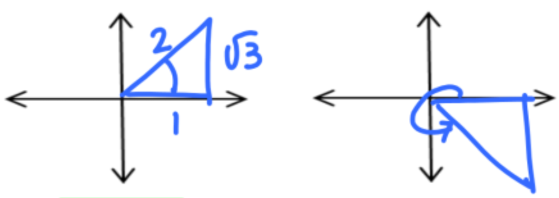
$$\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$$

$$\cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} - \left(\cancel{\sin x\cos\frac{\pi}{6}} - \cos x\sin\frac{\pi}{6}\right) = \frac{1}{2}$$

$$2\cos x\sin\frac{\pi}{6} = \frac{1}{2}$$

$$2\cos x\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\cos x = \frac{1}{2}$$

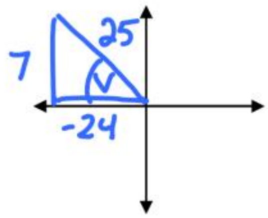


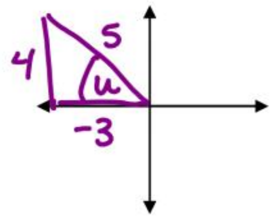
$$\boxed{x=\frac{\pi}{3}} \qquad \boxed{x=\frac{5\pi}{3}}$$

You try...

Given $\sin v = \frac{7}{25}$ and $\cos u = \frac{-3}{5}$.

Both angles ***u*** and **v** are in Quadrant II.
Find the exact value of each.





$$\sin(u+v) = \sin u \cos v + \cos u \sin v = \left(\frac{4}{5}\right)\left(-\frac{24}{25}\right) + \left(-\frac{3}{5}\right)\left(\frac{7}{25}\right) = \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{117}{125}}$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v = \left(\frac{-3}{5}\right)\left(\frac{-24}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{7}{25}\right) = \frac{72}{125} + \frac{28}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$= \frac{8}{8} \cdot \frac{\left(\frac{4}{-3}\right) - \left(\frac{7}{-24}\right)}{1 + \left(-\frac{4}{3}\right)\left(-\frac{7}{\cancel{24}_{6}}\right)} = \frac{-\frac{32}{24} + \frac{7}{24}}{1 + \frac{7}{18}} = \frac{-\frac{25}{24}}{\frac{25}{18}}$$

$$= -\frac{25}{24} \cdot \frac{18}{25} = -\frac{18}{24} = \boxed{-\frac{3}{4}}$$

You try again...

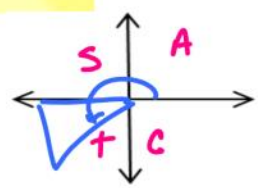
Verify the identity:

$$\cos\left(\frac{5\pi}{4} - x\right) = \frac{-\sqrt{2}}{2}(\cos x + \sin x)$$

$= \cos \frac{5\pi}{4} \cos x + \sin \frac{5\pi}{4} \sin x$

$= \left(-\frac{\sqrt{2}}{2}\right) \cos x + \left(-\frac{\sqrt{2}}{2}\right) \sin x$

$= \left(-\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)$ ✓



$\theta' = \frac{\pi}{4}$

One more time...

Find all the solutions in the interval $[0, 2\pi)$:

$$\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$$

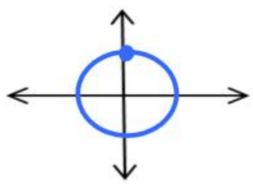
$$\sin x \cos\frac{\pi}{3} + \cancel{\cos x \sin\frac{\pi}{3}} + \sin x \cos\frac{\pi}{3} - \cancel{\cos x \sin\frac{\pi}{3}} = 1$$

$$2\sin x \cos\frac{\pi}{3} = 1$$

$$2\sin x\left(\frac{1}{2}\right) = 1$$

$$\sin x = 1$$

$$x = \boxed{\frac{\pi}{2}}$$



Using sum & differences to verify identities

$$\underline{\cos(x+y)} + \underline{\cos(x-y)} = 2\underline{\cos x \cos y}$$

$= \cos x \cos y - \cancel{\sin x \sin y} + \cos x \cos y + \cancel{\sin x \sin y}$

$= 2\cos x \cos y$ ✓

If time permits...

In Exercises 31–36, find the exact value of the expression.

31. $\sin 330^\circ \cos 30^\circ - \cos 330^\circ \sin 30^\circ$

33. $\sin \frac{\pi}{12} \cos \frac{\pi}{4} + \cos \frac{\pi}{12} \sin \frac{\pi}{4}$

In Exercises 37–44, find the exact value of the trigonometric function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II.)

37. $\sin(u + v)$ **38.** $\cos(u - v)$ **41.** $\tan(u + v)$

In Exercises 45–50, find the exact value of the trigonometric function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. (Both u and v are in Quadrant III.)

47. $\tan(u - v)$