Tuesday, February 26, 2019
5:41 PM

KEY

Precalc 5.4A: Sum & Dif. Formulas

Obj: To apply the sum & difference formulas to evaluate trig functions

Hwk: 5.4A #1, 9, 15, 25, 29 *FOLLOW DIRECTIONS!!!
5.4 Quiz Monday

Do Now:

1. Simplify:

a. $\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$ b. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ c. $\frac{\frac{\sqrt{3}}{3}-\frac{3}{4}}{1+\frac{\sqrt{3}}{3}\left(\frac{3}{4}\right)}$

2. Copy EXACTLY into notebooks:

Sum and Difference Formulas

$$\sin(u+v)=\sin u\cos v+\cos u\sin v$$

$$\sin(u-v)=\sin u\cos v-\cos u\sin v$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v$$

$$\cos(u-v)=\cos u\cos v+\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v} \qquad \tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

Do Now:

1. Simplify:

a. $\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b. $\frac{(1+\sqrt{3})}{(1-\sqrt{3})}\cdot\frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{1+\sqrt{3}+\sqrt{3}+3}{1+\sqrt{3}-\sqrt{3}-3} = \frac{4+2\sqrt{3}}{-2}$

* MULT BY conjugate of denominator

$= \frac{4}{-2}+\frac{2\sqrt{3}}{-2} = \boxed{-2-\sqrt{3}}$

* clear fraction

c. $\frac{\frac{12}{1}\left(\frac{\sqrt{3}}{3}-\frac{3}{4}\right)}{\frac{12}{1}\left(1+\frac{\sqrt{3}}{3}\left(\frac{3}{4}\right)\right)} = \frac{(4\sqrt{3}-9)}{(12+3\sqrt{3})}\cdot\frac{(12-3\sqrt{3})}{(12-3\sqrt{3})}$

$= \frac{48\sqrt{3}-12(3)-108+27\sqrt{3}}{144-9(3)}$

$= \boxed{\frac{75\sqrt{3}-144}{117}}$

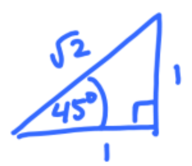
Class Notes:

Life is "messy" – it concerns more than 30°-60°-90° and 45°-45°-90° right triangles and quadrantal angles. These formulas help us to deal with non-standard angles.

Evaluate the following:

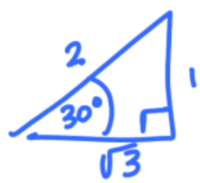
$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

Ex.1) $\cos(75°) = \cos(45°+\underline{30}°) = \cos(45°)\cos(\underline{30}°) - \sin(45°)\sin(\underline{30}°)$



$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$



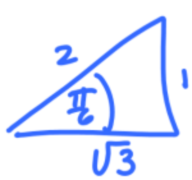
$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\frac{3\pi}{12}-\frac{2\pi}{12}$

Ex.2) $\sin\frac{\pi}{12} = \sin\left(\frac{3}{3}\cdot\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$



$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Ex.3) $\tan(255°)=$

$$\tan(300-45°) = \frac{\tan 300° - \tan 45°}{1+\tan 300° \tan 45°} = \frac{-\sqrt{3}-1}{1+(-\sqrt{3})(1)}$$

$$= \frac{(-1-\sqrt{3})}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{-1-2\sqrt{3}-3}{1-3} = \frac{-4-2\sqrt{3}}{-2} =$$

$$= \frac{-4}{-2} + \frac{-2\sqrt{3}}{-2} = \boxed{2+\sqrt{3}}$$

More importantly – how to remember the formulas since YOU MUST MEMORIZE THEM?

MIX (trig functions) and MATCH (signs) or
MATCH (trig functions) and MIX (signs)

Sine: MIX (trig functions) and MATCH (signs)
Cosine: MATCH (trig functions) and MIX (signs)
Tangent: MATCH (trig functions & signs) and MIX (signs)

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Working in reverse:

Write the expression as the sine, cosine, or tangent of a single angle. Find the exact value if possible. (dif. than worksheet)

Ex. 4) $\cos\left(\frac{7\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right)+\sin\left(\frac{7\pi}{9}\right)\sin\left(\frac{4\pi}{9}\right) = \cos(u-v)$

$= \cos\left(\frac{7\pi}{9}-\frac{4\pi}{9}\right)$

$= \cos\left(\frac{3\pi}{9}\right)$

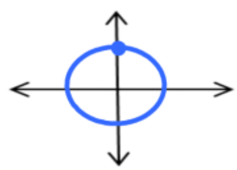
$= \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$

Ex. 5) $\sin 20°\cos 70° + \cos 20°\sin 70° = \sin(u+v)$

$= \sin(20° + 70°)$

$= \sin(90°)$

$= \boxed{1}$



Ex. 6) $\frac{\tan 80^{o} - \tan 50^{o}}{1+\tan 80^{o}\tan 50^{o}} = \tan(u-v)$

$= \tan(80° - 50°)$

$= \tan(30°)$

$= \boxed{\frac{\sqrt{3}}{3}}$

Do #4 - 9 from worksheet

Practice Problems:

NOTE: It is YOUR job to break up the angles into special angles that you can evaluate.

1. $\sin(105°)$

$= \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

2. $\cos(195°)$

$= \cos(45° + 150°)$

$= \cos 45° \cos 150° - \sin 45° \sin 150°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

3. $\tan\left(\frac{7\pi}{12}\right)$ * See work below

$$3.\ \tan\left(\frac{7\pi}{12}\right) = \tan\left(\underset{\frac{3\pi}{12}}{\frac{\pi}{4}} + \underset{\frac{4\pi}{12}}{\frac{\pi}{3}}\right)$$

$$= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{3}} = \frac{1+\sqrt{3}}{1-1(\sqrt{3})} = \frac{(1+\sqrt{3})}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$$

$$= \frac{1+2\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{-2} = \frac{4}{-2} + \frac{2\sqrt{3}}{-2}$$

$$= \boxed{-2-\sqrt{3}}$$