

Monday, March 05, 2018
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Name: KEY Date: _____ Period: _____
 Precalculus: 5.4 Quiz Review

1. Given $\sin u = \frac{7}{25}$ & $\cos v = \frac{3}{5}$. Angle u is in Quad. II and Angle v is in Quad. IV. Find the exact value of each:

a. $\sin(u - v)$

$$\begin{aligned} &\sin u \cos v - \cos u \sin v \\ &= \left(\frac{7}{25}\right)\left(\frac{3}{5}\right) - \left(\frac{-4}{5}\right)\left(-\frac{1}{5}\right) \\ &= \frac{21}{125} - \frac{4}{125} = \frac{-7}{125} = \boxed{\frac{-7}{125}} \end{aligned}$$

b. $\cos(u - v)$

$$\begin{aligned} &\cos u \cos v + \sin u \sin v \\ &= \left(\frac{-7}{25}\right)\left(\frac{3}{5}\right) + \left(\frac{7}{25}\right)\left(-\frac{4}{5}\right) \\ &= \frac{-21}{125} + \frac{-28}{125} \\ &= \frac{-100}{125} = \boxed{\frac{-4}{5}} \end{aligned}$$

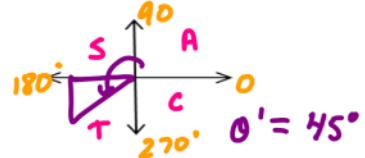
c. $\tan(u - v)$

$$\begin{aligned} &\frac{\tan u - \tan v}{1 - \tan u \tan v} \\ &= \frac{\frac{7}{24} - \left(-\frac{4}{3}\right)}{1 + \left(\frac{7}{24}\right)\left(-\frac{4}{3}\right)} = \frac{\frac{25}{24}}{\frac{25}{18}} \\ &= \frac{25}{24} \cdot \frac{18}{25} = \boxed{\frac{3}{4}} \end{aligned}$$

2. Write the expression as the sine, cosine or tangent of a single angle and find the exact value.

$$\cos 261^\circ \cos 36^\circ + \sin 261^\circ \sin 36^\circ$$

$$\begin{aligned} \cos(261^\circ - 36^\circ) &= \cos 225^\circ \\ &= \boxed{-\frac{\sqrt{2}}{2}} \end{aligned}$$

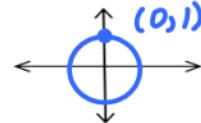


3. Verify: $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$$\cos \frac{\pi}{2} \cos x + \sin x \sin \frac{\pi}{2} = \sin x$$

$$(0) \cos x + \sin x (1) = \sin x$$

$$\sin x = \sin x \quad \checkmark$$



4. Solve over $[0, 2\pi]$: $\underline{\sin\left(x + \frac{\pi}{3}\right)} + \underline{\sin\left(x - \frac{\pi}{3}\right)} = 1$

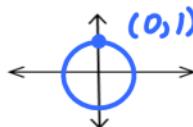
$$\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} + \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} = 1$$

$$\sin x \left(\frac{1}{2}\right) + \cos x \left(\frac{\sqrt{3}}{2}\right) + \sin x \left(\frac{1}{2}\right) - \cos x \left(\frac{\sqrt{3}}{2}\right) = 1$$

$$\cancel{\frac{1}{2}} \sin x + \cancel{\frac{1}{2}} \sin x = 1$$

$$\sin x = 1$$

$$\boxed{x = \frac{\pi}{2}}$$



5. Find the exact value of each using a sum or difference formula:

a. $\sin 255^\circ$

$$\begin{aligned}\sin(45^\circ + 210^\circ) &= \\ \sin 45^\circ \cos 210^\circ + \cos 45^\circ \sin 210^\circ &= \\ = (\frac{\sqrt{2}}{2})(-\frac{\sqrt{3}}{2}) + (\frac{\sqrt{2}}{2})(-\frac{1}{2}) &= \end{aligned}$$

$$\begin{aligned} &= -\frac{\sqrt{6}}{4} + -\frac{\sqrt{2}}{4} \\ &= \boxed{-\frac{\sqrt{6} - \sqrt{2}}{4}} \end{aligned}$$

b. $\cos 255^\circ$

$$\begin{aligned}\cos(45^\circ + 210^\circ) &= \\ \cos 45^\circ \cos 210^\circ - \sin 45^\circ \sin 210^\circ &= \\ = (\frac{\sqrt{2}}{2})(-\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2})(-\frac{1}{2}) &= \\ = -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} &= \\ = \boxed{-\frac{\sqrt{6} + \sqrt{2}}{4}} &= \end{aligned}$$

$$\begin{aligned} c. \tan 255^\circ &= \tan(45^\circ + 210^\circ) \\ \frac{\tan 45^\circ + \tan 210^\circ}{1 - \tan 45^\circ \tan 210^\circ} &= \\ \frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)(\frac{\sqrt{3}}{3})} &= \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} \cdot \frac{3}{3} \\ = \frac{(3+\sqrt{3})}{(3-\sqrt{3})} \cdot \frac{(3+\sqrt{3})}{(3+\sqrt{3})} &= \frac{9+6\sqrt{3}+3}{9-3} \\ = \frac{12+6\sqrt{3}}{6} &= \\ = \boxed{2+\sqrt{3}} &= \end{aligned}$$

6. Write the expression as the sine, cosine or tangent of a single angle and find the exact value.

a. $\sin 50^\circ \cos 20^\circ - \cos 50^\circ \sin 20^\circ =$

$$\begin{aligned}\sin(50^\circ - 20^\circ) &= \\ \sin 30^\circ &= \boxed{\frac{1}{2}}\end{aligned}$$

b. $\cos 40^\circ \cos 20^\circ - \sin 40^\circ \sin 20^\circ =$

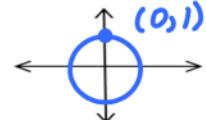
$$\begin{aligned}\cos(40^\circ + 20^\circ) &= \\ \cos 60^\circ &= \boxed{\frac{1}{2}}\end{aligned}$$

7. Verify: $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$$\sin \frac{\pi}{2} \cos x + \cos \frac{\pi}{2} \sin x = \cos x$$

$$(1) \cos x + (0) \sin x = \cos x$$

$$\cos x = \cos x \quad \checkmark$$



8. Solve over $[0, 2\pi)$: $\underline{\sin(x + \frac{\pi}{3})} + \underline{\sin(x - \frac{\pi}{3})} = \frac{1}{2}$

$$\cancel{\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}} + \cancel{\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3}} = \frac{1}{2}$$

$$2 \sin x \cos \frac{\pi}{3} = \frac{1}{2}$$

$$2 \sin x (\frac{1}{2}) = \frac{1}{2}$$

$$\sin x = \frac{1}{2} \quad \frac{S(A)}{\pi/6}$$

