

Thursday, February 22, 2018
8:12 PM

KEY

PreCalc

5.3E - Solving Multiple Angles Trig Equations

SHOW ALL WORK IN YOUR NOTEBOOK!

Solve. Give the general solution for each.

1. $\sin \frac{x}{2} = 0$

2. $\sec 4x = 2$

3. $\tan \frac{x}{3} = 1$

4. $\tan^2 3\theta = 3$

5. $3 \tan \frac{\alpha}{2} + 3 = 0$

Solve over $[0, 2\pi)$. Round to 4 decimal places if necessary.

6. $2\sin x + \cos x = 0; [0, 2\pi)$

7. $\tan^2 x - 6\tan x + 5 = 0$

8. $\tan^2 x + 3\tan x + 1 = 0$

Answers:

1. $x = 2\pi n, n$ is integer

2. $x = \frac{\pi}{12} + \frac{\pi n}{2}, x = \frac{5\pi}{12} + \frac{\pi n}{2}, n$ is integer

3. $x = \frac{3\pi}{4} + 3\pi n, n$ is integer

4. $\theta = \frac{\pi}{9} + \frac{\pi n}{3}, \theta = \frac{2\pi}{9} + \frac{\pi n}{3}, n$ is integer

5. $\alpha = \frac{3\pi}{2} + 2\pi n, n$ is integer

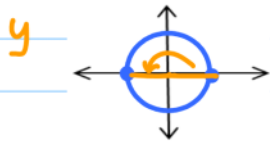
6. $x \approx 2.6779, 5.8195$

7. $x = \frac{\pi}{4}, \frac{5\pi}{4}; x \approx 1.3734, 4.5150$

8. $x \approx 2.7767, 5.9183, x \approx 1.9357, 5.0773$

Solve. Give the general solution for each.

1. $\sin \frac{x}{2} = 0$

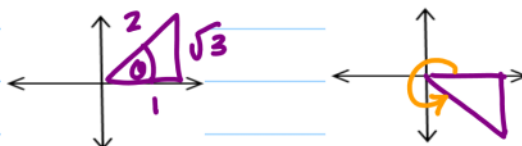


$2 \left(\frac{1}{2}x \right) = (0 + 2\pi n) \cdot 2$
 $2 \left(\frac{1}{2}x \right) = (\pi + 2\pi n) \cdot 2$

$x = 0 + 4\pi n$
 $x = 2\pi + 4\pi n$
 $n \in \mathbb{Z}$

* can combine to $x = 0 + 2\pi n$

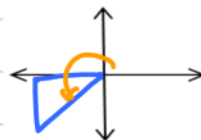
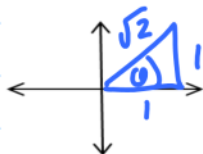
2. $\sec 4x = 2$



$\frac{1}{4} (4x) = \left(\frac{\pi}{3} + 2\pi n \right) \cdot \frac{1}{4}$
 $x = \frac{\pi}{12} + \frac{\pi n}{2}$

$\frac{1}{4} (4x) = \left(\frac{5\pi}{3} + 2\pi n \right) \cdot \frac{1}{4}$
 $x = \frac{5\pi}{12} + \frac{\pi n}{2}$
 $n \in \mathbb{Z}$

3. $\tan \frac{x}{3} = 1$



$3 \left(\frac{x}{3} \right) = \left(\frac{\pi}{4} + \pi n \right) \cdot 3$

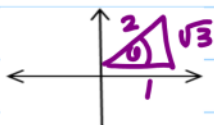
$3 \left(\frac{x}{3} \right) = \left(\frac{5\pi}{4} + \pi n \right) \cdot 3$

$x = \frac{3\pi}{4} + 3\pi n$
 $n \in \mathbb{Z}$

$x = \frac{15\pi}{4} + 3\pi n$
 * can combine with 1st answer

4. $\sqrt{\tan^2 3\theta} = \sqrt{3}$

$\tan 3\theta = \pm \sqrt{3}$



$\frac{1}{3} (3\theta) = \left(\frac{\pi}{3} + \pi n \right) \cdot \frac{1}{3}$
 $\theta = \frac{\pi}{9} + \frac{\pi n}{3}$

$\frac{1}{3} (3\theta) = \left(\frac{2\pi}{3} + \pi n \right) \cdot \frac{1}{3}$
 $\theta = \frac{2\pi}{9} + \frac{\pi n}{3} \cdot \frac{2}{3}$
 $n \in \mathbb{Z}$



$\frac{1}{3} (3\theta) = \left(\frac{4\pi}{3} + \pi n \right) \cdot \frac{1}{3}$
 $\theta = \frac{4\pi}{9} + \frac{\pi n}{3}$

$\frac{1}{3} (3\theta) = \left(\frac{5\pi}{3} + \pi n \right) \cdot \frac{1}{3}$
 $\theta = \frac{5\pi}{9} + \frac{\pi n}{3}$

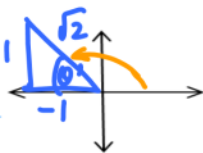
* can combine with other 2 answers

$$5. 3 \tan \frac{\alpha}{2} + 3 = 0$$

$$3 \tan \frac{\alpha}{2} = -3$$

$\frac{S}{A}$
 $\frac{T}{C}$

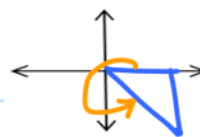
$$\tan \frac{\alpha}{2} = -1$$



$$2 \left(\frac{1}{2} \alpha \right) = \left(\frac{3\pi}{4} + \pi n \right) 2$$

$$\alpha = \frac{3\pi}{2} + 2\pi n$$

$$n \in \mathbb{Z}$$



$$2 \left(\frac{1}{2} \alpha \right) = \left(\frac{7\pi}{4} + \pi n \right) 2$$

$$\alpha = \frac{7\pi}{2} + 2\pi n$$

* Can combine with 1st answer

Solve over $[0, 2\pi)$. Round to 4 decimal places if necessary.

$$6. 2 \sin x + \cos x = 0; [0, 2\pi)$$

$$(2 \sin x)^2 = (-\cos x)^2$$

$$4 \sin^2 x = \cos^2 x$$

$$4 \sin^2 x - \cos^2 x = 0$$

* Pythag ID

$$4 \sin^2 x + \sin^2 x - 1 = 0$$

$$5 \sin^2 x - 1 = 0$$

$$5x^2 - 1 = 0 \quad a=5 \quad b=0 \quad c=-1$$

$$\sin x = \frac{-0 \pm \sqrt{0^2 - 4(5)(-1)}}{2(5)} = \frac{\pm \sqrt{20}}{10}$$

pos ↓

2(5)

10

neg ←

$\frac{S}{A}$
 $\frac{T}{C}$

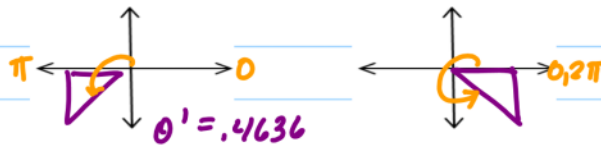
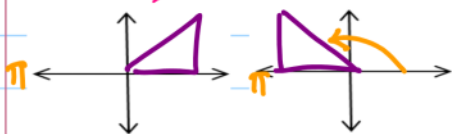
$$\sin^{-1} \left(\frac{\sqrt{20}}{10} \right) = x$$

$$\sin^{-1} \left(-\frac{\sqrt{20}}{10} \right) = x$$

$\frac{S}{A}$
 $\frac{T}{C}$

$$x = .4636 \quad * \text{extraneous}$$

$$x = -.4636$$



$$\pi - .4636 = 2.6780$$

$$\pi + .4636 = 3.6052$$

$$2\pi - .4636$$

* extraneous

$$= 5.8196$$

* MUST check for extraneous solutions!

$$2 \sin x + \cos x = 0$$

$$2 \sin(3.6052) + \cos(3.6052) = 0$$

$$2 \sin(.4636) + \cos(.4636) = 0$$

$$-1.7889 \neq 0$$

$$1.7888 \neq 0$$

$$7. \tan^2 x - 6 \tan x + 5 = 0$$

$$(\tan x - 5)(\tan x - 1) = 0$$

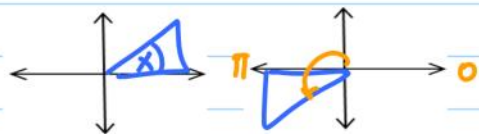
$$x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1) = 0$$

S/A
T/C

$$\tan x - 5 = 0$$

$$\tan x = 5$$



$$\tan^{-1}(5) = x$$

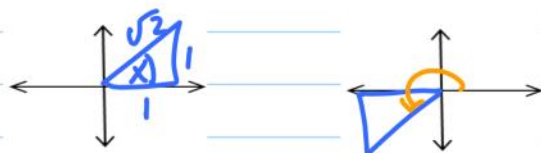
$$x = 1.3734$$

$$\pi + 1.3734$$

$$x = 4.5150$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$



$$x = \frac{\pi}{4}$$

$$x = \frac{5\pi}{4}$$

$$8. \tan^2 x + 3 \tan x + 1 = 0$$

$$x^2 + 3x + 1 = 0$$

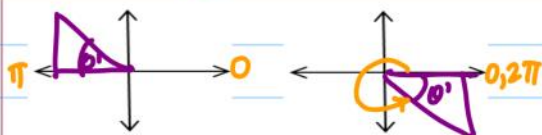
$$\tan x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

S/A
T/C

$$\tan x = \frac{-3 + \sqrt{5}}{2} \approx -0.3820$$

$$\tan^{-1}\left(\frac{-3 + \sqrt{5}}{2}\right) = x$$

$$x = -0.3649$$



$$x = \pi - 0.3649$$

$$x \approx 2.7767$$

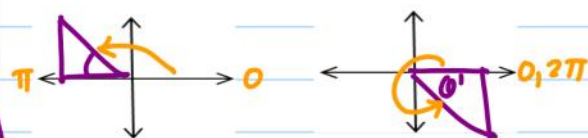
$$x = 2\pi - 0.3649$$

$$x \approx 5.9183$$

$$\tan x = \frac{-3 - \sqrt{5}}{2}$$

$$\tan^{-1}\left(\frac{-3 - \sqrt{5}}{2}\right) = x$$

$$x = -1.2059$$



$$x = \pi - 1.2059$$

$$x \approx 1.9357$$

$$x = 2\pi - 1.2059$$

$$x \approx 5.0773$$

S/A
T/C