Saturday, February 17, 2018
9:56 AM

KEY

5.3 B – Solving Trigonometric Equations

Homework:
- Section 5.3 B
- Check answers on Calc. Chat!

Objective:
Use standard algebraic techniques to solve trig equations

Do Now:

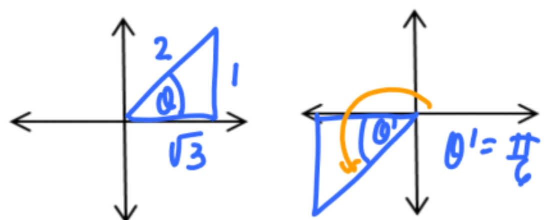
Solve the following trig equation on the interval [0,2π):

$$\sqrt{3}\tan x - 1 = 0$$

$\sqrt{3}\tan x = 1$

$\div$ $\tan x = \frac{1}{\sqrt{3}}$

S | A
T | C



$\boxed{\theta = \frac{\pi}{6}}$ $\boxed{\theta = \frac{7\pi}{6}}$

General Solution:

$$\boxed{\begin{array}{ll} \frac{\pi}{6} + \pi n & \\ \frac{7\pi}{6} + \pi n & n \in \mathbb{Z} \end{array}}$$

Homework Questions??

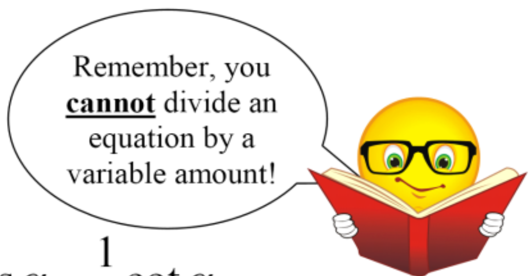
Classnotes....

To obtain general solutions. . .

- Add 2π to solutions involving sine, cosine, cosecant and secant.
- Add π to solutions involving tangent and cotangent.

Strategies to use when solving trig equations include. . .

- Isolating the trig function
- Taking the square root * use ±, 4 answers!
- Factoring
- Substituting trig identities
- Squaring Equations



Practice Problems....

1. Find the general solution of $\cot\alpha\cos\alpha = \frac{1}{2}\cot\alpha$.

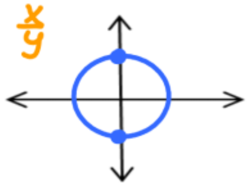
Hint: When your instinct tells you to divide by a function, set equal to zero and factor instead!

$\cot\alpha\cos\alpha - \frac{1}{2}\cot\alpha = 0$

$\cot\alpha(\cos\alpha - \frac{1}{2}) = 0$

$\cot\alpha = 0$

$\frac{x}{y}$



$\cos\alpha - \frac{1}{2} = 0$

$\cos\alpha = \frac{1}{2}$

S | A
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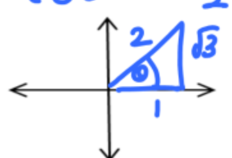
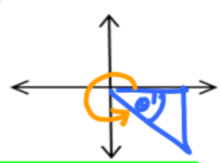




$\boxed{\theta = \frac{\pi}{2} + \pi n, \frac{3\pi}{2} + \pi n}$ $\boxed{\theta = \frac{\pi}{3} + 2\pi n}$ $\boxed{\theta = \frac{5\pi}{3} + 2\pi n}$ $n \in \mathbb{Z}$

Practice Problems....

2. Find all the solutions in the interval [0,2π) for:

$$2\sin^2 x - 3\sin x + 1 = 0.$$

$2x^2 - 3x + 1$

$(2x-1)(x-1)$

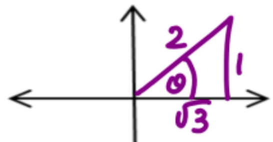
$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ | $\sin x - 1 = 0$

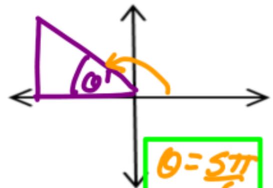
$2\sin x = 1$ | $\sin x = 1$

$\sin x = \frac{1}{2}$

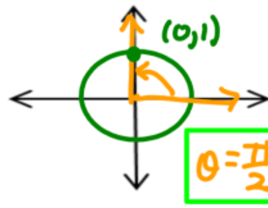
S A
T C



$\boxed{\theta = \frac{\pi}{6}}$



$\boxed{\theta = \frac{5\pi}{6}}$



$\boxed{\theta = \frac{\pi}{2}}$

Squaring to find solutions....

Find all solutions in the interval [0,2π).

Hint: Square both sides, FOIL & sub Pythag ID.

$$\sin\beta + 1 = \cos\beta.$$

$(\sin\beta + 1)(\sin\beta + 1) = \cos^2\beta$

$\sin^2\beta + 2\sin\beta + 1 = \cos^2\beta$

$\sin^2\beta + 2\sin\beta + \underline{1 - \cos^2\beta} = 0$

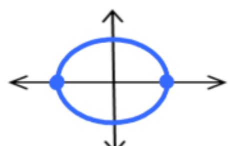
$\sin^2\beta + 2\sin\beta + \underline{\sin^2\beta} = 0$

$2\sin^2\beta + 2\sin\beta = 0$

$2\sin\beta(\sin\beta + 1) = 0$

$2\sin\beta = 0$

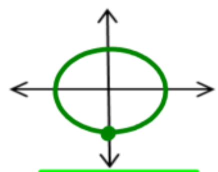
$\sin\beta = 0$



$\boxed{\cancel{\theta = \pi} \quad \theta = 0}$

$\sin\beta + 1 = 0$

$\sin\beta = -1$



$\boxed{\theta = \frac{3\pi}{2}}$

$\sin 0 + 1 = \cos 0$
$0 + 1 = 1$
$1 = 1$
$\beta = 0$ is a solution.

$\sin\pi + 1 = \cos\pi$
$0 + 1 = -1$
$1 \neq -1$
$\beta = \pi$ is an extraneous solution.

$\sin\frac{3\pi}{2} + 1 = \cos\frac{3\pi}{2}$
$-1 + 1 = 0$
$0 = 0$
$\beta = \frac{3\pi}{2}$ is a solution.

When squaring to solve, you MUST check for ***extraneous solutions*** *by substituting into the original equation to determine if it is a solution!* *(Remember, an extraneous solution is one that does not satisfy the original equation.)*

Practice Problems cont.

3. Find all solutions in the interval $[0,2\pi)$ of
$\sec x - 1 = \tan x$.

*Square both sides

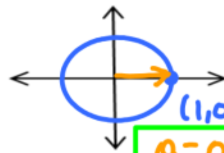
$(\sec x - 1)(\sec x - 1) = \tan^2 x$

$\sec^2 x - 2\sec x + 1 = \underline{\tan^2 x}$ *Pythag

$\sec^2 x - 2\sec x + 1 = \underline{\sec^2 x - 1}$

$-2\sec x + 2 = 0$

$-2\sec x = -2$ $\quad \sec x = 1$ $\quad \overset{+}{x}$



$\boxed{\theta = 0}$

Check: $\sec 0 - 1 = \tan 0$

$1 - 1 = 0 \quad 0 = 0$ ✓

Don't forget to check for extraneous solutions!

Practice Problems cont.

4. Find the general solution of $\sin y = \cos y$.

* Square both sides
* Pythag. I.D.

$\sin^2 y = \underline{\cos^2 y}$

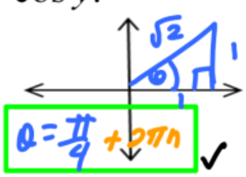
$\sin^2 y = 1 - \underline{\sin^2 y}$

$2\sin^2 y = 1$

$\sqrt{\sin^2 y} = \sqrt{\frac{1}{2}}$

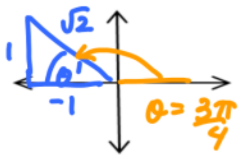
$|\sin y| = \frac{1}{\sqrt{2}}$

$\sin y = \pm \frac{1}{\sqrt{2}}$



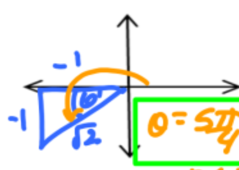
$\boxed{\theta = \frac{\pi}{4} + 2\pi n}$ ✓

Check: $\sin\frac{\pi}{4} = \cos\frac{\pi}{4}$



$\theta = \frac{3\pi}{4}$

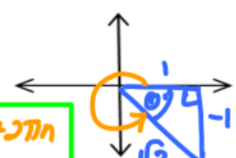
$\sin\frac{3\pi}{4} \neq \cos\frac{3\pi}{4}$
not a solution



$\boxed{\theta = \frac{5\pi}{4} + 2\pi n}$

$n \in \mathbb{Z}$ ✓

Check: $\sin\frac{5\pi}{4} = \cos\frac{5\pi}{4}$



$\theta = \frac{7\pi}{4}$

Check: $\sin\frac{7\pi}{4} = \cos\frac{7\pi}{4}$
not a solution

Don't forget to check for extraneous solutions!

Example of no solution

Find the general solution of $2\cos\alpha + \sec\alpha = 0$.

* clear fraction

$\cos\alpha\left(2\cos\alpha + \frac{1}{\cos\alpha}\right) = (0)\cos\alpha$

$2\cos^2\alpha + 1 = 0$

$2\cos^2\alpha = -1$

$\sqrt{\cos^2\alpha} = \sqrt{-\frac{1}{2}}$

even root

negative radicand

$\boxed{\text{no solution}}$