

Monday, February 12, 2018

6:23 PM

KEY

Trig Identities Worksheet 5.2

Prove Each Identity.

$$\begin{aligned}
 1. \quad \frac{1 + \cos \beta}{\sin \beta} &= \underline{\csc \beta + \cot \beta} \\
 &= \frac{1}{\sin \beta} + \frac{\cos \beta}{\sin \beta} \\
 &= \underline{\csc \beta + \cot \beta} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} &= \underline{\sin^2 \theta} \\
 &= \frac{1 \cdot \sin \theta}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta \cos \theta}{1} = \underline{\sin^2 \theta} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{\sec A}{\cos A} - \frac{\tan A}{\cot A} &= 1 \\
 &= \frac{\sec A}{\frac{1}{\sec A}} - \frac{\tan A}{\frac{1}{\tan A}} \\
 &= \sec A \cdot \sec A - \tan A \tan A \\
 &= \sec^2 A - \tan^2 A = \underline{1} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \csc^2 x \tan^2 x - 1 &= \underline{\tan^2 x} \\
 &= \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} - 1 \\
 &= \frac{1}{\cos^2 x} - 1 \cdot \frac{\cos^2 x}{\cos^2 x} \\
 &= \frac{1 - \cos^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} = \underline{\tan^2 x} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{\sec^2 y}{\sec^2 y - 1} &= \underline{\csc^2 y} \\
 &= \frac{\sec^2 y}{\tan^2 y} \\
 &= \frac{1}{\frac{\cos^2 y}{\sin^2 y}} = \frac{1}{\cancel{\cos^2 y}} \cdot \frac{\cancel{\cos^2 y}}{\sin^2 y} \\
 &= \frac{1}{\sin^2 y} = \underline{\csc^2 y} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \underline{\tan^2 \alpha} \sin^2 \alpha &= \tan^2 \alpha - \sin^2 \alpha \\
 &= \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta}{1} \cdot \frac{\cos^2 \theta}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta (\sin^2 \theta)}{\cos^2 \theta} = \underline{\tan^2 \theta \sin^2 \theta} \quad \checkmark
 \end{aligned}$$

Trig Identities Worksheet 5.2

Prove Each Identity.

$$7. \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} = 1 - 2 \sin^2 \alpha$$

$$\frac{\cos^2 \alpha \frac{1 - \sin^2 \alpha}{\cos^2 \alpha}}{\cos^2 \alpha \frac{1 + \sin^2 \alpha}{\cos^2 \alpha}} = \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha + \sin^2 \alpha}$$

$$= \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha} \cdot \frac{\cos^2 \alpha}{1} = 1 - \sin^2 \alpha - \sin^2 \alpha$$

$$= 1 - 2 \sin^2 \alpha \quad \checkmark$$

$$8. \frac{\cos A + 1}{\sin^3 A} = \frac{\csc A}{1 - \cos A} \cdot \frac{(1 + \cos A)}{(1 + \cos A)}$$

$$= \frac{\csc A (1 + \cos A)}{\sin^2 A}$$

$$= \frac{1}{\sin A} \cdot \frac{(1 + \cos A)}{\sin^2 A} = \frac{\cos A + 1}{\sin^3 A} \quad \checkmark$$

$$9. (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$= (\sin \theta + \cos \theta)(\sin \theta + \cos \theta) + (\sin \theta - \cos \theta)(\sin \theta - \cos \theta)$$

$$= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta$$

$$= \sin^2 \theta + \cos^2 \theta + \sin^2 \theta + \cos^2 \theta$$

$$= 1 + 1 = 2 \quad \checkmark$$

$$10. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$= (\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x)$$

$$= \csc^2 x + \cot^2 x (1)$$

$$= \csc^2 x + \cot^2 x \quad \checkmark$$

$$11. \frac{\tan \beta}{\sec \beta} + \frac{\cot \beta}{\csc \beta} = \sin \beta + \cos \beta$$

$$\frac{\sec \beta}{\sec \beta} = \frac{1}{\csc \beta} + \frac{1}{\sec \beta} \cdot \frac{\csc \beta}{\csc \beta}$$

$$= \frac{\sec \beta}{\sec \beta \csc \beta} + \frac{\csc \beta}{\sec \beta \csc \beta}$$

$$= \frac{1}{\cos \beta} \cdot \frac{1}{\sin \beta} + \frac{1}{\sin \beta} \cdot \frac{1}{\cos \beta} \cdot \csc \beta$$

$$= \frac{\sin \beta}{\cos \beta} + \frac{\cos \beta}{\sin \beta} = \frac{\tan \beta}{\sec \beta} + \frac{\cot \beta}{\csc \beta} \quad \checkmark$$

$$12. \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$

$$\frac{\cos y}{\cos y} \frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}} = \frac{\cos y \sin y + \sin y}{\cos y + 1}$$

$$= \frac{\sin y (\cos y + 1)}{\cos y + 1} \cdot \frac{\cos y}{\cos y} = \sin y \quad \checkmark$$