Friday, February 08, 2019
7:38 AM

Precalc Beginning Identities Name KEY

Prove each of the following using the established rules.
Work on **one side** only (either side will do). Do not skip steps - if in doubt, put it in.

1. $\cot\beta = \cos\beta\csc\beta$

2. $\cos x \sec x = \tan x \cot x$

3. $\dfrac{1+\cot^2\alpha}{1+\tan^2\alpha} = \cot^2\alpha$

4. $(\sec\theta + 1)(\sec\theta - 1) = \tan^2\theta$

5. $(\sin^2\theta - \cos^2\theta) = 2\sin^2\theta - 1$

6. $(\sin\beta - \cos\beta)^2 = 1 - 2\sin\beta\cos\beta$

7. $\dfrac{\tan x - \cos x}{\sin x} = \sec x - \cot x$

8. $(\sec\theta + 1)(\cos\theta - 1) = \cos\theta - \sec\theta$

9. $\dfrac{\tan\beta - \cot\beta}{\tan\beta + \cot\beta} = 1 - 2\cos^2\beta$

10. $\dfrac{1}{1-\cos\theta} - \dfrac{1}{1+\cos\theta} = 2\cot\theta\csc\theta$

11. $\dfrac{1+2\tan\alpha}{1-2\tan\alpha} = \dfrac{\cot\alpha + 2}{\cot\alpha - 2}$

12. $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

13. $\dfrac{\cot x + 1}{\cot x - 1} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

14. $\tan^2\theta + \csc^2\theta = \cot^2\theta + \sec^2\theta$

15. $\sec\alpha - \cos\alpha = \sin\alpha\tan\alpha$

!sdrawkcab kniht ot evah uoy semitemoS

1. $\cot\beta = \cos\beta \csc\beta$

$= \cos\beta \cdot \frac{1}{\sin\beta}$

$= \frac{\cos\beta}{\sin\beta}$

$= \cot\beta$ ✓

2. $\cos x \sec x = \tan x \cot x$

$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos x$

$= \frac{1}{\cos x} \cdot \cos x$

$= \sec x \cos x = \cos x \sec x$ ✓

3. $\frac{1+\cot^2\alpha}{1+\tan^2\alpha} = \cot^2\alpha$

$= \frac{\csc^2\alpha}{\sec^2\alpha}$

$= \frac{\frac{1}{\sin^2\alpha}}{\frac{1}{\cos^2\alpha}} = \frac{1}{\sin^2\alpha} \cdot \frac{\cos^2\alpha}{1}$

$= \cot^2\alpha$ ✓

4. $(\sec\theta + 1)(\sec\theta - 1) = \tan^2\theta$

$= \sec^2\theta - 1$

$= \tan^2\theta$ ✓

5. $(\sin^2\theta - \cos^2\theta) = 2\sin^2\theta - 1$

$= \sin^2\theta + \sin^2\theta - 1$

$= 1 - \cos^2\theta + \sin^2\theta - 1$

$= \sin^2\theta - \cos^2\theta$ ✓

* Pythag. I.D.

6. $(\sin\beta - \cos\beta)^2 = 1 - 2\sin\beta\cos\beta$

$= (\sin\beta - \cos\beta)(\sin\beta - \cos\beta)$

$= \sin^2\beta - 2\sin\beta\cos\beta + \cos^2\beta$

$= \sin^2\beta + \cos^2\beta - 2\sin\beta\cos\beta$

$= 1 - 2\sin\beta\cos\beta$ ✓

* Alternative method below

7. $\frac{\tan x - \cos x}{\sin x} = \sec x - \cot x$

$= \frac{\tan x}{\sin x} - \frac{\cos x}{\sin x}$

$= \frac{\frac{\sin x}{\cos x}}{\sin x} - \cot x$

$= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} - \cot x = \frac{1}{\cos x} - \cot x$ ✓

$= \sec x - \cot x$

8. $(\sec\theta + 1)(\cos\theta - 1) = \cos\theta - \sec\theta$

$\sec\theta\cos\theta - \sec\theta + \cos\theta - 1 =$

$\frac{1}{\cos\theta} \cdot \cos\theta - \sec\theta + \cos\theta - 1 =$

$1 - \sec\theta + \cos\theta - 1 =$

$\cos\theta - \sec\theta$ ✓

9. $\frac{\tan\beta - \cot\beta}{\tan\beta + \cot\beta} = 1 - 2\cos^2\beta$ ✓

$= \frac{\frac{\sin\beta}{\cos\beta} - \frac{\cos\beta}{\sin\beta}}{\frac{\sin\beta}{\cos\beta} + \frac{\cos\beta}{\sin\beta}} \cdot \frac{\sin\beta\cos\beta}{\sin\beta\cos\beta}$

$= \frac{\sin^2\beta - \cos^2\beta}{\sin^2\beta + \cos^2\beta}$

$= \sin^2\beta - \cos^2\beta \cdot \frac{\sin\beta\cos\beta}{\sin^2\beta + \cos^2\beta}$

$= \frac{\sin^2\beta - \cos^2\beta}{1} = 1 - \cos^2\beta - \cos^2\beta = 1 - 2\cos^2\beta$ ✓

* pythag. ID

10. $\frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta} = 2\cot\theta\csc\theta$

$\frac{(1+\cos\theta)}{(1+\cos\theta)} \cdot \frac{(1-\cos\theta)}{(1-\cos\theta)}$

$\frac{1+\cos\theta - (1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} =$

$\frac{2\cos\theta}{1 - \cos^2\theta} = \frac{2\cos\theta}{\sin^2\theta}$

$= 2\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}$

$= 2\cot\theta\csc\theta$ ✓

* Alternative method:

⑦ $\frac{\tan x - \cos x}{\sin x} = \sec x - \cot x$

$= \frac{\sin x}{\sin x \cos x} \perp - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$

$= \frac{\sin x - \cos^2 x}{\sin x \cos x}$

$= \frac{\cos x \left(\frac{\sin x}{\cos x} - \cos x\right)}{\sin x \cos x}$

$= \frac{\tan x - \cos x}{\sin x}$ ✓

KEY

11. $\dfrac{1+2\tan\alpha}{1-2\tan\alpha} = \dfrac{\cot\alpha+2}{\cot\alpha-2}$

$= \dfrac{\frac{1}{\tan\alpha}+2 \cdot \frac{\tan\alpha}{\tan\alpha}}{\frac{1}{\tan\alpha}-2 \cdot \frac{\tan\alpha}{\tan\alpha}} = \dfrac{\frac{1}{\tan\alpha}+\frac{2\tan\alpha}{\tan\alpha}}{\frac{1}{\tan\alpha}-\frac{2\tan\alpha}{\tan\alpha}} = \dfrac{\frac{1+2\tan\alpha}{\tan\alpha}}{\frac{1-2\tan\alpha}{\tan\alpha}}$

$= \dfrac{1+2\tan\alpha}{\tan\alpha} \cdot \dfrac{\tan\alpha}{1-2\tan\alpha} = \dfrac{1+2\tan\alpha}{1-2\tan\alpha}$ ✓

12. $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$= \dfrac{\sec^2\theta}{\sec^2\theta} - \dfrac{1}{\sec^2\theta} = 1-\cos^2\theta = \sin^2\theta$ ✓

* pythag ID

13. $\dfrac{\cot x+1}{\cot x-1} = \dfrac{\cos x+\sin x}{\cos x-\sin x}$

$\dfrac{\frac{\cos x}{\sin x}+1 \cdot \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}-1 \cdot \frac{\sin x}{\sin x}} = \dfrac{\frac{\cos x+\sin x}{\sin x}}{\frac{\cos x-\sin x}{\sin x}} = \dfrac{\cos x+\sin x}{\sin x} \cdot \dfrac{\sin x}{\cos x-\sin x}$

$= \dfrac{\cos x+\sin x}{\cos x-\sin x}$ ✓

* Pythag. ID

14. $\tan^2\theta + \csc^2\theta = \cot^2\theta + \sec^2\theta$

$= \sec^2\theta - 1 + \csc^2\theta$

$= \sec^2\theta - 1 + 1 + \cot^2\theta = \cot^2\theta + \sec^2\theta$ ✓

15. $\sec\alpha - \cos\alpha = \sin\alpha\tan\alpha$ ✓

$= \frac{1}{\cos\alpha} - \frac{\cos\alpha}{1} \cdot \frac{\cos\alpha}{\cos\alpha}$

$= \frac{1-\cos^2\alpha}{\cos\alpha} = \frac{\sin^2\alpha}{\cos\alpha} = \sin\alpha \cdot \frac{\sin\alpha}{\cos\alpha} = \sin\alpha\tan\alpha$ ✓