Monday, February 26, 2018
5:50 PM

Precalc Practice Trig. Equations/Identities Name KEY

I. Solve the following over the interval $\left[0, 2\pi\right)$ unless otherwise indicated. *calculator necessary

(answers supplied on the reverse side)

1. $\sqrt{2}\cos x + 1 = 0$
2. Give ***all*** solutions to: $\sqrt{3}\sec x + 2 = 0$
3. $2\sin^3 x - \sin x = 0$
4. $1 - \sin x - \cos x = 0$
5. $3\sin^2 x - \cos^2 x = 0$

*6. $\sec^2 x - 2\tan x = 4$

II. Prove:

7. $\dfrac{\csc\theta - \sec\theta}{\csc\theta + \sec\theta} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$
8. $(\cos\theta - \cot\theta)(\sec\theta + \tan\theta) = \sin\theta - \csc\theta$

Answer:

1. $\left\{\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right\}$
2. $\left\{\dfrac{5\pi}{6} + 2\pi n,\ \dfrac{7\pi}{6} + 2\pi n;\ \text{where } n \text{ is an integer}\right\}$
3. $\left\{0, \pi, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$
4. $\left\{0, \dfrac{\pi}{2}\right\}$
5. $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}$
6. $\left\{1.249,\ 4.391,\ \dfrac{3\pi}{4} \text{ or } 2.356, \dfrac{7\pi}{4} \text{ or } 5.498\right\}$

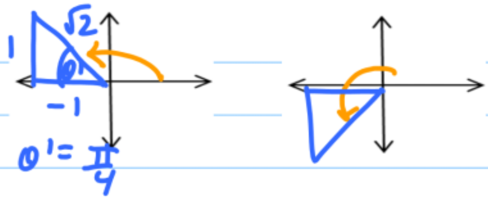
I. Solve the following over the interval $[0, 2\pi)$ unless otherwise indicated. *calculator necessary

1. $\sqrt{2}\cos x + 1 = 0$

S | A
T | C

$\sqrt{2}\cos x = -1$

$\cos x = -\frac{1}{\sqrt{2}}$



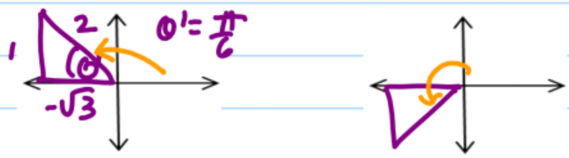
$\boxed{x = \frac{3\pi}{4}}$ $\boxed{x = \frac{5\pi}{4}}$

2. Give ***all*** solutions to: $\sqrt{3}\sec x + 2 = 0$

S | A
T | C

$\sqrt{3}\sec x = -2$

$\frac{h}{a}$ $\sec x = -\frac{2}{\sqrt{3}}$

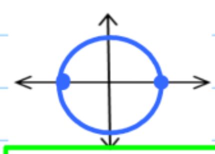


$\boxed{x = \frac{5\pi}{6} + 2\pi n}$ $\boxed{x = \frac{7\pi}{6} + 2\pi n}$ $\boxed{n \in \mathbb{Z}}$

3. $2\sin^3 x - \sin x = 0$

$\sin x(2\sin^2 x - 1) = 0$

$\sin x = 0$



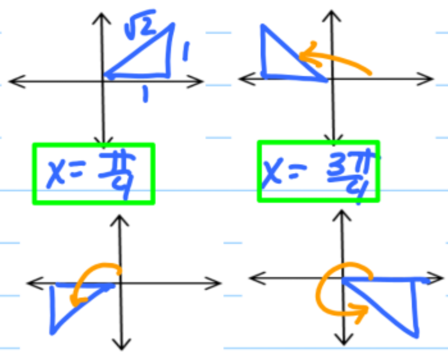
$\boxed{x = 0 \qquad x = \pi}$

$2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}}$



$\boxed{x = \frac{\pi}{4}}$ $\boxed{x = \frac{3\pi}{4}}$ $\boxed{x = \frac{5\pi}{4}}$ $\boxed{x = \frac{7\pi}{4}}$

4. $1 - \sin x - \cos x = 0$

$(1 - \sin x)^2 = (\cos x)^2$

$(1 - \sin x)(1 - \sin x) = \cos^2 x$

$1 - 2\sin x + \sin^2 x = \cos^2 x$ * Pythag. ID

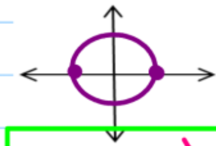
$1 - 2\sin x + \sin^2 x = 1 - \sin^2 x$

$2\sin^2 x - 2\sin x = 0$

$2\sin x(\sin x - 1) = 0$

$2\sin x = 0$

$\sin x = 0$



$\boxed{x = 0 \quad \cancel{x = \pi}}$

extraneous

Check: $1 - \sin 0 - \cos 0 = 0$

$1 - 0 - 1 = 0 \qquad 0 = 0 \checkmark$

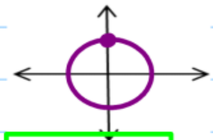
$1 - \sin\pi - \cos\pi = 0$

$1 - 0 - -1 = 0$

$2 \neq 0$

$\sin x - 1 = 0$

$\sin x = 1$



$\boxed{x = \frac{\pi}{2}}$

Check:

$1 - \sin x - \cos x = 0$

$1 - \sin\frac{\pi}{2} - \cos\frac{\pi}{2} = 0$

$1 - 1 - 0 = 0$

$0 = 0 \checkmark$

5. $3\sin^2 x - \underline{\cos^2 x} = 0$

$3\sin^2 x - (\underline{1 - \sin^2 x}) = 0$ * Pythag ID

$3\sin^2 x - 1 + \sin^2 x = 0$

$4\sin^2 x - 1 = 0$

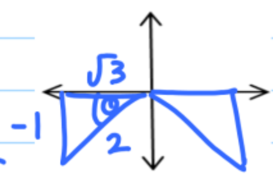
$(2\sin x + 1)(2\sin x - 1) = 0$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

S | A

T | C

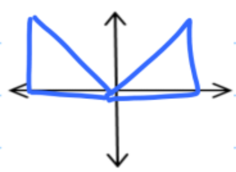


$\theta' = \frac{\pi}{6}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$



$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

*6. $\sec^2 x - 2\tan x = 4$ * Pythag. ID

$1+\tan^2 x - 2\tan x = 4$

$\tan^2 x - 2\tan x - 3 = 0$ $x^2 - 2x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$ $(x-3)(x+1) = 0$

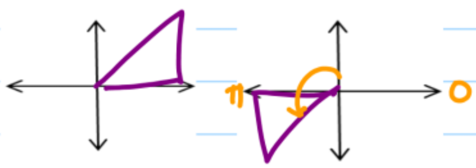
$\tan x - 3 = 0$ $\tan x + 1 = 0$

$\tan x = 3$ $\tan x = -1$

$\tan^{-1}(3) = x$

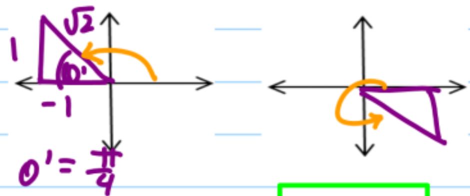
$\boxed{x \approx 1.2490}$

S | A
T | C



$x = \pi + 1.2490$

$\boxed{x \approx 4.3906}$



$\theta' = \frac{\pi}{4}$

$\boxed{x = \frac{3\pi}{4}}$ $\boxed{x = \frac{7\pi}{4}}$

II. Prove: 7. $\dfrac{\csc\theta - \sec\theta}{\csc\theta + \sec\theta} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$$\frac{\frac{\cos\theta}{\cos\theta}\cdot\left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right)\cdot\frac{\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\cos\theta}\cdot\left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)\cdot\frac{\sin\theta}{\sin\theta}} =$$

$$\frac{\frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}}{\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}} =$$

$$\frac{\cos\theta - \sin\theta}{\cancel{\sin\theta}\,\cancel{\cos\theta}} \cdot \frac{\cancel{\sin\theta}\,\cancel{\cos\theta}}{\cos + \sin\theta} =$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} \checkmark$$

8. $(\cos\theta - \cot\theta)(\sec\theta + \tan\theta) = \sin\theta - \csc\theta$

$$\left(\cos\theta - \frac{1}{\tan\theta}\right)\left(\frac{1}{\cos\theta} + \tan\theta\right) =$$ "FOIL"

$$\frac{\cos\theta}{\cos\theta} + \cos\theta\tan\theta - \frac{1}{\tan\theta\cos\theta} - \frac{\tan\theta}{\tan\theta} =$$

$$1 + \cos\theta\tan\theta - \frac{1}{\tan\theta\cos\theta} - 1 =$$

$$\cos\theta\tan\theta - \cot\theta\sec\theta =$$

$$\cancel{\cos\theta}\frac{\sin\theta}{\cancel{\cos\theta}} - \frac{\cancel{\cos\theta}}{\sin\theta}\cdot\frac{1}{\cancel{\cos\theta}} =$$

$$\sin\theta - \frac{1}{\sin\theta} =$$

$$\sin\theta - \csc\theta \checkmark$$