

Monday, February 26, 2018

5:51 PM

KEY

Sections 5.2, 5.3 Review Exercises, #25, 29, 33 - 49 odd  
Show all work on a SEPARATE sheet of paper

**5.2** In Exercises 25–32, verify the identity.

25.  $\cos x(\tan^2 x + 1) = \sec x$   
26.  $\sec^2 x \cot x - \cot x = \tan x$   
27.  $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$   
28.  $\cot\left(\frac{\pi}{2} - x\right) = \tan x$   
29.  $\frac{1}{\tan \theta \csc \theta} = \cos \theta$   
30.  $\frac{1}{\tan x \csc x \sin x} = \cot x$   
31.  $\sin^5 x \cos^2 x = (\cos^2 x - 2 \cos^4 x + \cos^6 x) \sin x$   
32.  $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x) \cos x$

**5.3** In Exercises 33–38, solve the equation.

33.  $\sin x = \sqrt{3} - \sin x$       34.  $4 \cos \theta = 1 + 2 \cos \theta$   
35.  $3\sqrt{3} \tan u = 3$   
36.  $\frac{1}{2} \sec x - 1 = 0$   
37.  $3 \csc^2 x = 4$   
38.  $4 \tan^2 u - 1 = \tan^2 u$

In Exercises 39–46, find all solutions of the equation in the interval  $[0, 2\pi)$ .

39.  $2 \cos^2 x - \cos x = 1$       42.  $\sin^2 x + 2 \cos x = 2$   
40.  $2 \sin^2 x - 3 \sin x = -1$   
41.  $\cos^2 x + \sin x = 1$       44.  $\sqrt{3} \tan 3x = 0$   
43.  $2 \sin 2x - \sqrt{2} = 0$       46.  $3 \csc^2 5x = -4$   
45.  $\cos 4x(\cos x - 1) = 0$

In Exercises 47–50, use inverse functions where needed to find all solutions of the equation in the interval  $[0, 2\pi)$ .

47.  $\sin^2 x - 2 \sin x = 0$       48.  $2 \cos^2 x + 3 \cos x = 0$   
49.  $\tan^2 \theta + \tan \theta - 12 = 0$

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5.2 In Exercises 25–32, verify the identity.

25.  $\cos x(\tan^2 x + 1) = \sec x$

$\cos x(\sec^2 x) =$   
 $\cos x \left( \frac{1}{\cos^2 x} \right) =$

$\frac{1}{\cos x} =$

$\sec x$  ✓

29.  $\frac{1}{\tan \theta \csc \theta} = \cos \theta$

$\frac{1}{\tan \theta} \cdot \frac{1}{\csc \theta} =$

$\frac{1}{\frac{\sin \theta}{\cos \theta}} \cdot \frac{1}{\frac{1}{\sin \theta}} =$

$1 \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{1} = \cos \theta$  ✓

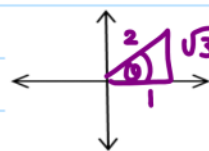
5.3 In Exercises 33–38, solve the equation.

33.  $\sin x = \sqrt{3} - \sin x$

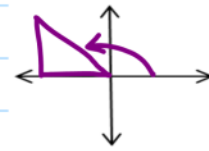
$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\frac{S}{A} \frac{C}{C}$



$x = \frac{\pi}{3} + 2\pi n$



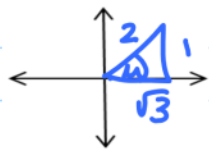
$x = \frac{2\pi}{3} + 2\pi n$

35.  $3\sqrt{3} \tan u = 3$

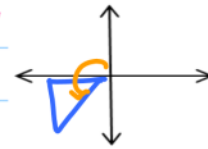
$\tan u = \frac{3}{3\sqrt{3}}$

$\tan u = \frac{1}{\sqrt{3}}$

$\frac{S}{A} \frac{C}{C}$



\*  $u = \frac{\pi}{6} + \pi n$



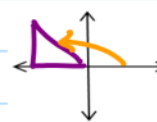
$u = \frac{7\pi}{6} + \pi n$

\* you can combine both answers to  $\frac{\pi}{6} + \pi n$  because if you add  $\frac{\pi}{6} + \frac{4\pi}{6}$  you get  $\frac{5\pi}{6}$ .

37.  $3 \csc^2 x = 4$

$\sqrt{\csc^2 x} = \sqrt{\frac{4}{3}}$

$\csc x = \pm \frac{2}{\sqrt{3}}$



\*  $x = \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

\* these can be combined as:  $x = \frac{\pi}{3} + \pi n$  and  $\frac{2\pi}{3} + \pi n$

In Exercises 39–46, find all solutions of the equation in the interval  $[0, 2\pi)$ .

39.  $2 \cos^2 x - \cos x = 1$

$$2 \cos^2 x - \cos x - 1 = 0 \qquad 2x^2 - x - 1 = 0$$

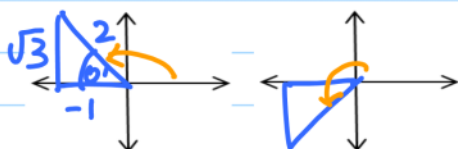
$$(2 \cos x + 1)(\cos x - 1) = 0 \qquad (2x + 1)(x - 1) = 0$$

$$2 \cos x + 1 = 0 \qquad \cos x - 1 = 0$$

$$2 \cos x = -1 \qquad \cos x = 1$$

$$\cos x = -\frac{1}{2}$$

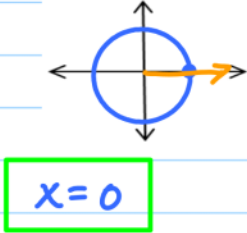
~~S/A~~  
~~A/C~~



$$\theta' = \frac{\pi}{3}$$

$$x = \frac{2\pi}{3}$$

$$x = \frac{4\pi}{3}$$



$$x = 0$$

41.  $\cos^2 x + \sin x = 1$

\* Pythag ID

$$1 - \sin^2 x + \sin x = 1$$

$$-\sin^2 x + \sin x = 0$$

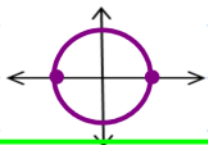
$$-\sin x (\sin x - 1) = 0$$

$$-\sin x = 0$$

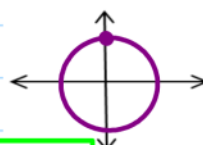
$$\sin x = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$



$$x = 0, x = \pi$$

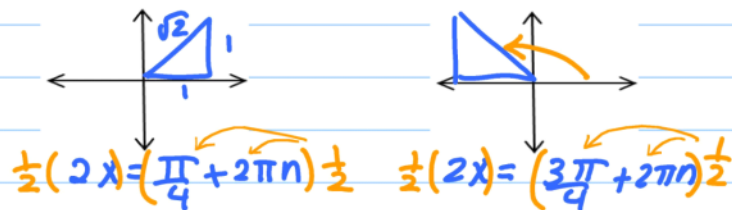


$$x = \frac{\pi}{2}$$

43.  $2 \sin 2x - \sqrt{2} = 0$

$2 \sin 2x = \sqrt{2}$   
 $\sin 2x = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin 2x = \frac{1}{\sqrt{2}}$   $\frac{S}{1A}$   
 $\frac{T}{C}$



$\frac{1}{2}(2x) = \left(\frac{\pi}{4} + 2\pi n\right) \frac{1}{2}$      $\frac{1}{2}(2x) = \left(\frac{3\pi}{4} + 2\pi n\right) \frac{1}{2}$

\*  $x = \frac{\pi}{8} + \pi n$     \*  $x = \frac{3\pi}{8} + \pi n$

\* general solutions

All solutions in the interval  $[0, 2\pi)$ :

$\frac{\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{11\pi}{8}$

"  $\frac{\pi}{8} + \frac{8\pi}{8}$     "  $\frac{3\pi}{8} + \frac{8\pi}{8}$

\* ADD  $\pi$  TO GET OTHER SOLUTIONS UP TO  $2\pi$

45.  $\cos 4x(\cos x - 1) = 0$

$\cos 4x = 0$

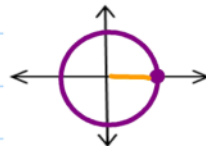


$\frac{1}{4}(4x) = \left(\frac{\pi}{2} + 2\pi n\right) \frac{1}{4}$

\*  $x = \frac{\pi}{8} + \frac{\pi}{2} n$

$\cos x - 1 = 0$

$\cos x = 1$



\*  $x = 0 + 2\pi n$

\* general solutions

$\frac{1}{4}(4x) = \left(\frac{3\pi}{2} + 2\pi n\right) \frac{1}{4}$

\*  $x = \frac{3\pi}{8} + \frac{\pi}{2} n$

All solutions in the interval  $[0, 2\pi)$ :

$0, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Keep adding  $\frac{\pi}{8}$  to  $\frac{\pi}{8}$  until you get all solutions up to  $2\pi$

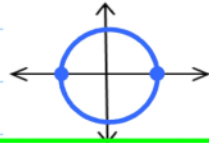
Keep adding  $\frac{\pi}{8}$  to  $\frac{3\pi}{8}$  until you get all solutions up to  $2\pi$ .

In Exercises 47–50, use inverse functions where needed to find all solutions of the equation in the interval  $[0, 2\pi)$ .

47.  $\sin^2 x - 2 \sin x = 0$

$\sin x (\sin x - 2) = 0$

$\sin x = 0$



$x = 0, x = \pi$

$\sin x - 2 = 0$

$\sin x = 2$

$\sin^{-1}(2) = x \leftarrow$  Domain error

NO solution

\* 2 is not in the range of  $\sin x$

49.  $\tan^2 \theta + \tan \theta - 12 = 0$

$(\tan x + 4)(\tan x - 3) = 0$

$\tan x + 4 = 0$

$\tan x = -4 \leftarrow$  neg

$\tan^{-1}(-4) = x$

$x = -1.3258$

$\frac{S}{A}$   
 $\frac{T}{C}$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$\tan x - 3 = 0$

$\tan x = 3 \leftarrow$  pos

$\tan^{-1}(3) = x$

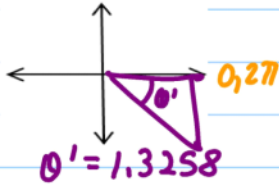
$x = 1.2490$

$\frac{S}{A}$   
 $\frac{T}{C}$



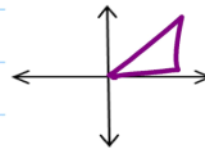
$\pi - 1.3258$

$x = 1.8158$

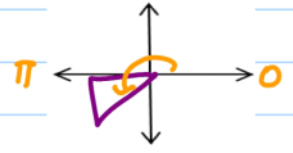


$2\pi - 1.3258$

$x = 4.9574$



$x = 1.2490$



$\pi + 1.2490$

$x = 4.3906$

\* Equivalent to text answer

$\theta = \arctan(-4) + n\pi$

$\theta = \arctan 3 + n\pi$

$\theta = \arctan(-4) + \pi, \arctan(-4) + 2\pi, \arctan 3, \arctan 3 + \pi$