Tuesday, February 06, 2018
7:41 PM

Precalc **5.1B Simplifying Trig Expressions** **Name:** KEY

Factor the following algebra expressions:

#1 $x^2y^2 + x^4$ GCF!
$x^2(y^2+x^2)$

#2 $zy^2 + z$
$z(y^2+1)$

#3 $x^4 + 2x^2 + 1$
$(x^2+1)(x^2+1)$

For each of the following trigonometry expressions:

1) **Factor. (Follow one of the three patterns above.)**
2) **After you factor, simplify using trig identities.**

$\sin^2\theta\cos^2\theta + \sin^4\theta$

$= \sin^2\theta\,(\underbrace{\cos^2\theta + \sin^2\theta}_{\text{Pythag. Identity}})$

$= \sin^2\theta\,(1)$

$= \boxed{\sin^2\theta}$

$\csc x + \cot^2 x\csc x$

$= \csc x\,(\underbrace{1 + \cot^2 x}_{\text{Pythag. Identity}})$

$= \csc x\,(\csc^2 x)$

$= \boxed{\csc^3 x}$

* FACTOR the Trinomial

$\cot^4 y + 2\cot^2 y + 1$

$= (\cot^2 y + 1)(\cot^2 y + 1)$

$= (\csc^2 y)(\csc^2 y)$

$= \boxed{\csc^4 y}$

* Pythag. ID
$1+\cot^2 y = \csc^2 y$

Use algebra techniques to simplify (factoring, getting common denominators to add fractions, and so on).

#4 $\dfrac{1-x^2}{1-x}$

$\dfrac{(1-x)(1+x)}{(1-x)}$

$= \boxed{1+x}$

#5 $\dfrac{(1+y)}{(1+y)}\cdot\dfrac{1}{1-y} + \dfrac{1}{1+y}\cdot\dfrac{(1-y)}{(1-y)}$

$\dfrac{1+y}{(1+y)(1-y)} + \dfrac{1-y}{(1+y)(1-y)}$

$= \boxed{\dfrac{2}{(1+y)(1-y)}}$

#6 $z^3 - 4z^2 - 5z + 20$

$z^2(z-4) - 5(z-4)$

$\boxed{(z^2-5)(z-4)}$

#7 $\dfrac{(1+x)}{1+x}\cdot\dfrac{1+x}{y} + \dfrac{y}{1+x}\cdot\dfrac{y}{y}$

$\dfrac{1+2x+x^2}{y(1+x)} + \dfrac{y^2}{y(1+x)} =$

$\boxed{\dfrac{x^2+2x+1+y^2}{y(1+x)}}$

Simplify the following using trig identities. Use the same algebra steps you used for problems #4 to #7.

$\dfrac{\cos^2\alpha}{1-\sin\alpha} = \dfrac{1-\sin\alpha}{1-\sin\alpha}$

* Pythag. Id
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$

$= \boxed{1}$

$\dfrac{(1-\sin\beta)}{(1-\sin\beta)}\cdot\dfrac{1}{1+\sin\beta} + \dfrac{1}{1-\sin\beta}\cdot\dfrac{1+\sin\beta}{1+\sin\beta} = \dfrac{1-\sin\beta}{(1-\sin\beta)(1+\sin\beta)} + \dfrac{1+\sin\beta}{(1-\sin\beta)(1+\sin\beta)}$

$= \dfrac{1-\sin\beta+1+\sin\beta}{(1-\sin\beta)(1+\sin\beta)} = \dfrac{2}{1-\sin^2\beta}$ * FOIL

$= \dfrac{2}{\cos^2\beta} = 2\,\dfrac{1}{\cos^2\beta} = \boxed{2\sec^2\beta}$

$\sec^3 x - \sec^2 x - \sec x + 1 =$

$= \sec^2 x(\sec x - 1) - 1(\sec x - 1)$

* $(\sec^2 x - 1)(\sec x - 1)$

$= (\tan^2 x)(\sec x - 1)$

* Pythag. Identity

$\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x}{1+\cos x} =$

$= \dfrac{(1+\cos x)(1+\cos x)}{(1+\cos x)(\sin x)} + \dfrac{\sin^2 x}{(1+\cos x)(\sin x)}$

$= \dfrac{1+\cos x+\cos x+\cos^2 x}{(1+\cos x)(\sin x)} + \dfrac{\sin^2 x}{(1+\cos x)(\sin x)}$

$= \dfrac{1+2\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)(\sin x)}$

$$= \frac{1 + 2\cos x + 1}{(1+\cos x)(\sin x)} = \frac{2\cos x + 2}{(1+\cos x)(\sin x)}$$

$$= \frac{2\cancel{(\cos x + 1)}}{(1+\cancel{\cos x})(\sin x)} = 2 \cdot \frac{1}{\sin x} = \boxed{2\csc x}$$

Simplify the following:

$$\frac{\sec^2\theta - 1}{\sec\theta - 1} = \frac{(\sec\theta + 1)\cancel{(\sec\theta - 1)}}{\cancel{(\sec\theta - 1)}}$$

$$= \boxed{\sec\theta + 1}$$

$$(\cos\beta - \sin\beta)^2$$

$$= (\cos\beta - \sin\beta)(\cos\beta - \sin\beta)$$

$$= \underline{\cos^2\beta} - 2\sin\beta\cos\beta + \underline{\sin^2\beta}$$

$$= \boxed{1 - 2\sin\beta\cos\beta}$$

$$\sin^2 x\csc^2 x - \sin^2 x$$ * Pythag. ID

$$= \sin^2 x(\csc^2 x - 1)$$

$$= \sin^2 x\,(\underline{\cot^2 x})$$

$$= \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$$

$$= \boxed{\cos^2 x}$$

$$1 - 2\cos^2 x + \cos^4 x$$

$$\cos^4 x - 2\cos^2 x + 1 =$$

$$(\cos^2 x - 1)(\cos^2 x - 1) =$$

$$(-\sin^2 x)(-\sin^2 x) =$$

$$\boxed{\sin^4 x}$$

* Pythag. ID
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta - 1 = -\sin^2\theta$

$$(\cot x + \csc x)(\cot x - \csc x)$$

$$= \underline{\cot^2 x - \csc^2 x}$$

$$= \boxed{-1}$$

* Pythagorean Identity

$$\frac{\tan x}{\tan x} \cdot \tan x - \frac{\sec^2 x}{\tan x} \qquad \frac{\tan^2 x}{\tan x} - \frac{\sec^2 x}{\tan x}$$

$$= \frac{\underline{\tan^2 x - \sec^2 x}}{\tan x} = \frac{-1}{\tan x} = \boxed{-\cot x}$$

$$\sec^4 x - \tan^4 x$$

$$= (\underline{\sec^2 x - \tan^2 x})(\sec^2 x + \tan^2 x)$$

$$= (1)(\sec^2 x + \tan^2 x)$$

$$= \boxed{\sec^2 x + \tan^2 x}$$

$$\frac{\sec x - 1}{\sec x - 1} \cdot \frac{1}{\sec x + 1} - \frac{1}{\sec x - 1} \cdot \frac{\sec x + 1}{\sec x + 1}$$

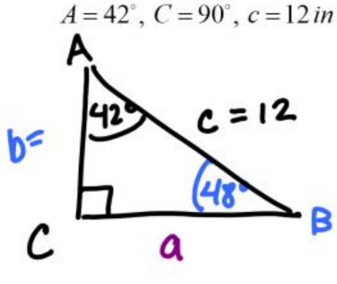
$$\frac{\sec x - 1}{(\sec x + 1)(\sec x - 1)} - \frac{\sec x + 1}{(\sec x + 1)(\sec x - 1)} =$$

$$\frac{-2}{\underline{\sec^2 x - 1}} = \frac{-2}{\tan^2 x} = \boxed{-2\cot^2 x}$$

* pythag. ID

Solve the following triangle. Sketch and label each part and show all work.

$A = 42^\circ$, $C = 90^\circ$, $c = 12\,in$



$90 - 42^\circ = 48^\circ$

$\sin 42 = \frac{a}{12}$

$12\sin 42^\circ = a$

$a = 8.03''$

$\cos 42^\circ = \frac{b}{12}$

$12\cos 42^\circ = b$

$b = 8.92''$

$A = 42^\circ$	$a \approx 8.03''$
$B = 48^\circ$	$b \approx 8.92''$
$C = 90^\circ$	$c = 12\,in$