Wednesday, January 30, 2019
6:11 PM

KEY

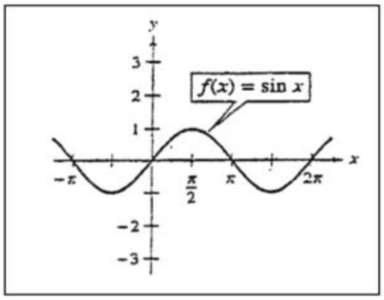
Precalc 5.1A: Using Identities

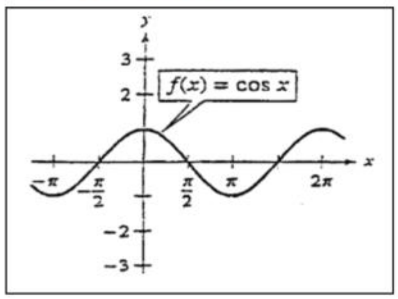
Obj: To apply the fundamental trig identities to simplify trig expressions;

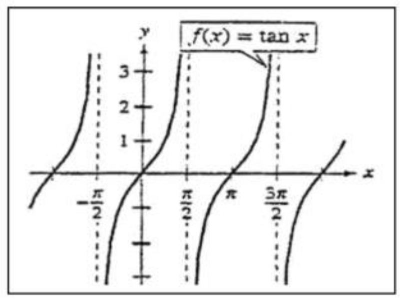
Hwk: 5.1A #15-19, 21-25, 27-35 (odds), 39, 41, 43, Check answers!!!

Do Now:

Look at the graphs of the sine, cosine and tangent functions. Which functions are even? Odd?







ODD

* Symmetric about x-Axis + y-Axis

Even

* symmetric over y-Axis

ODD

* Symmetric about x-Axis + y-Axis

Fundamental Trig Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Six *reciprocal* identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{\frac{o}{h}}{\frac{a}{h}} = \frac{o}{h}\cdot\frac{h}{a} = \frac{o}{a}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Two *Quotient* Identities

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Three *Pythagorean* Identities

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Fundamental Trig Identities

Cofunction Identities * cofunctions add to 90° (complementary angles)

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Six *Cofunction* Identities

Even/Odd Identities

ODD $\quad \sin(-\mu) = -\sin\mu \qquad \csc(-\mu) = -\csc\mu$

Even $\quad \cos(-\mu) = \cos\mu \qquad \sec(-\mu) = \sec\mu$

ODD $\quad \tan(-\mu) = -\tan\mu \qquad \cot(-\mu) = -\cot\mu$

Six *Even/Odd* Identities

Use the Fundamental Trig identities to simplify the expression:

1) $\sec\theta\cot\theta$

$\frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta} = \frac{1}{\sin\theta}$

$= \boxed{\csc\theta}$

2) $\sin\theta\csc\theta$

$\frac{1}{\csc\theta} \cdot \csc\theta = \frac{\csc\theta}{\csc\theta} = \boxed{1}$

3) $\tan\theta\cos\theta$

$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta} = \boxed{\sin\theta}$

4) $\cot\theta\sin\theta$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \boxed{\cos\theta}$

5) * $(1-\sin^2 x)(\sec^2 x)$

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$(\cos^2 x)(\sec^2 x) =$

$\cancel{\cos^2 x} \cdot \frac{1}{\cancel{\cos^2 x}} = \boxed{1}$

6) $(1-\cos^2 x)(\csc x)$

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$(\sin^2 x)(\csc x) =$

$\sin^2 x \cdot \frac{1}{\sin x} = \frac{\sin^2 x}{\sin x}$

$= \boxed{\sin x}$

7) $\sin x\cos^2 x - \sin x$

FACTOR OUT GCF!

$\sin x(\cos^2 x - 1)$

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - 1 = -\sin^2\theta$

$= \sin x(-\sin^2 x)$

$= \boxed{-\sin^3 x}$

8) $\csc(-\beta)\sin\beta$

$\frac{1}{\sin(-\beta)} \cdot \sin\beta = \frac{\sin\beta}{\sin(-\beta)}$

* Even/ODD Identity

$= \frac{\sin\beta}{-\sin\beta} = \boxed{-1}$

Use the Fundamental Trig identities to simplify the expression:

9) $\dfrac{\sin^2 y}{1-\cos y} = \dfrac{1-\cos^2 y}{1-\cos y}$

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$= \dfrac{(1+\cos y)\cancel{(1-\cos y)}}{\cancel{(1-\cos y)}}$

$= \boxed{1+\cos y}$

10) $\dfrac{\tan^2 \phi}{\sec^2 \phi} = \dfrac{\frac{\sin\phi}{\cos\phi}}{\frac{1}{\cos\phi}}$

$= \dfrac{\sin\phi}{\cancel{\cos\phi}} \cdot \dfrac{\cancel{\cos\phi}}{1} = \boxed{\sin\phi}$

11) $\csc\theta\tan\theta + \sec\theta$

$= \left(\dfrac{1}{\cancel{\sin\theta}}\right)\left(\dfrac{\cancel{\sin\theta}}{\cos\theta}\right) + \sec\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{1}{\cos\theta}$

$= \dfrac{2}{\cos\theta} = 2\left(\dfrac{1}{\cos\theta}\right)$

$= \boxed{2\sec\theta}$

12) $\cos\beta\cot\beta + \sin\beta$

$= \cos\beta\left(\dfrac{\cos\beta}{\sin\beta}\right) + \sin\beta$

$= \dfrac{\cos^2\beta}{\sin\beta} + \sin\beta \cdot \dfrac{\sin\beta}{\sin\beta}$

$= \dfrac{\cos^2\beta}{\sin\beta} + \dfrac{\sin^2\beta}{\sin\beta}$

$= {}^{*}\dfrac{\cos^2\beta + \sin^2\beta}{\sin\beta} = \dfrac{1}{\sin\beta}$

* Pythagorean Identity

$= \boxed{\csc\beta}$

Your turn.........

Use the Fundamental Trig identities to simplify the expression:

13) $\cos\beta\sec\beta$

$\cos\beta \cdot \frac{1}{\cos\beta} = \frac{\cos\beta}{\cos\beta} = \boxed{1}$

14) $\frac{\csc^2\theta - 1}{\cot\theta}$

* Pythagorean Identity

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\frac{\cot^2\theta}{\cot\theta} = \boxed{\cot\theta}$

15) $\cos\phi\left(\frac{\tan\phi}{\sin\phi}\right)$

$= \cos\phi\left(\frac{\frac{\sin\phi}{\cos\phi}}{\sin\phi}\right)$

$= \cancel{\cos}\phi \cdot \frac{\cancel{\sin}\phi}{\cancel{\cos}\phi} \cdot \frac{1}{\cancel{\sin}\phi}$

$= \boxed{1}$

Recap:

Basic ideas: ***Have your formulas out when doing cw/hw!***

1. Look for identities with same functions and substitute
2. Factor if necessary