Tuesday, January 08, 2019
4:26 PM

KEY

Precalc 4.7C: Inverse Trig Functions

Obj: to eval. inverse trig functions & composition of inverse trig functions; to apply props. of inverse trig functions

Hwk: Finish "4.7 Inverse Trig Functions" worksheet

4.7 Performance Assessment – FRIDAY

Do Now:

1. Recap:

   a. State the domain & range for each inverse trig function:

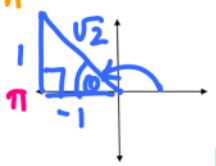
   $\sin^{-1}(x)$: D: $[-1, 1]$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

   $\cos^{-1}(x)$: D: $[-1, 1]$ R: $[0, \pi]$

   $\tan^{-1}(x)$: D: $(-\infty, \infty)$ R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

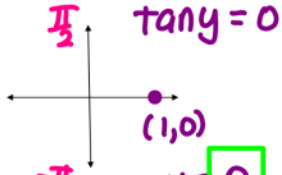
2. Find the value of each of the following:

a. arc cos $\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{\sqrt{2}}{\sqrt{2}}$ $-\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$



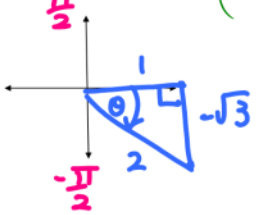
$\theta' = \frac{\pi}{4}$ $y = \frac{3\pi}{4}$

b. arc tan 0



$y = 0$

c. arc sin $\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$



$\theta' = \frac{\pi}{3}$

d. $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$? ✓

$= -\frac{\pi}{6}$

e. $\cos\left(\cos^{-1}\frac{1}{9}\right)$ $[-1, 1]$? ✓

$= \frac{1}{9}$

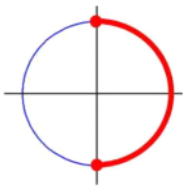
f. $\tan\left(\arctan\left(-\frac{\sqrt{3}}{3}\right)\right)$ $(-\infty, \infty)$? ✓

$= -\frac{\sqrt{3}}{3}$

Recap:

$y = \arcsin x$ or $y = \sin^{-1} x$

Def: $y = \arcsin x$ iff $\sin y = x$

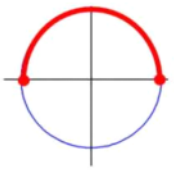


Domain : $[-1,1]$

Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \arccos x$ or $y = \cos^{-1} x$

Def: $y = \arccos x$ iff

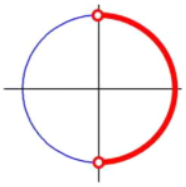


Domain : $[-1,1]$

Range : $[0, \pi]$

$y = \arctan x$ or $y = \tan^{-1} x$

Def: $y = \arctan x$ iff $\tan y = x$



Domain : $(-\infty, \infty)$

Range : $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

If you see **"arc" or $\sin^{-1}(x)$**, give the **ANGLE MEASURE** (in **RADIANS**).

- Check domain, then draw triangle in appropriate quadrant & find angle w/ given trig value

Inverse properties of Trig functions: (when trig ratios/ functions are the same)

- if $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ then

  sin (arc sin x) = x and arc sin (sin y) = y

- if $-1 \le x \le 1$ and $0 \le y \le \pi$ then

  cos (arc cos x) = x and arc cos (cos y) = y

- if $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$ then

  tan (arc tan x) = x and arc tan (tan y) = y

*If evaluating a TRIG FUNCTION, your answer is a RATIO

*If evaluating an INVERSE TRIG FUNCTION, your answer is an ANGLE

Examples:

1. $\sin \arcsin\left(\frac{2}{3}\right)$ $[-1,1]$? ✓ $= \boxed{\frac{2}{3}}$

2. $\sin \arcsin\left(\frac{3}{2}\right)$ $[-1,1]$? NO $\boxed{\text{NOT possible, not in domain.}}$

3. $\cos \arccos(.7)$ $[-1,1]$? ✓ $= \boxed{.7}$

4. $\cos\left(\cos^{-1} 7\right)$ $[-1,1]$? NO $\boxed{\text{NOT possible, not in domain.}}$

5. $\tan \arctan(100)$ $(-\infty,\infty)$? ✓ $= \boxed{100}$

6. $\tan\left(\tan^{-1} \pi\right)$ $(-\infty,\infty)$? ✓ $= \boxed{\pi}$

The inverse properties only apply to ***DEFINED*** values of x & y.
(i.e. in correct domain or range)

e.g. $\arcsin\left(\sin\frac{3\pi}{2}\right) = \frac{3\pi}{2}$ right? NO!

REWRITE in allowed range with equivalent angle:

$$\arcsin\left(\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2} \qquad \underline{\text{OR}}$$

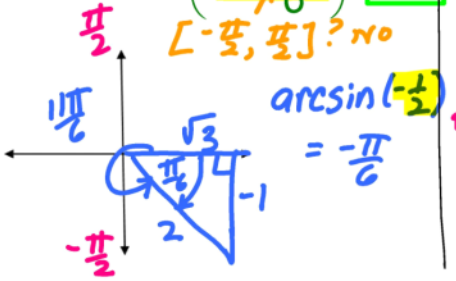
EVALUTE inside function first

$\sin\frac{3\pi}{2} = -1$ so $\arcsin(-1) = -\frac{\pi}{2}$

If trig ratios/functions are same, check domain/range:

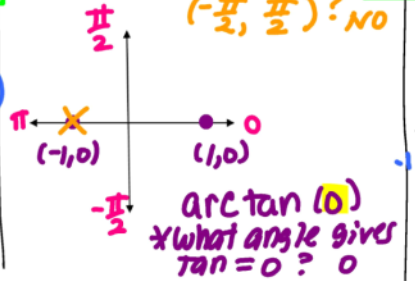
Examples:

7. $\arcsin\left(\sin\frac{11\pi}{6}\right) = \boxed{-\frac{\pi}{6}}$ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$? NO



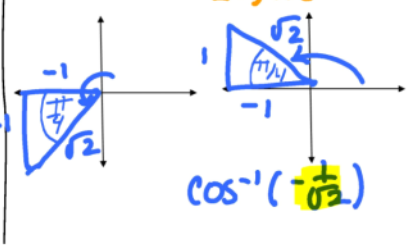
$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

8. $\arctan(\tan\pi) = \boxed{0}$ $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$? NO



$\arctan(0)$ *what angle gives $\tan = 0$? 0

9. $\cos^{-1}\left(\cos\frac{5\pi}{4}\right) = \boxed{\frac{3\pi}{4}}$ $[0,\pi]$? NO

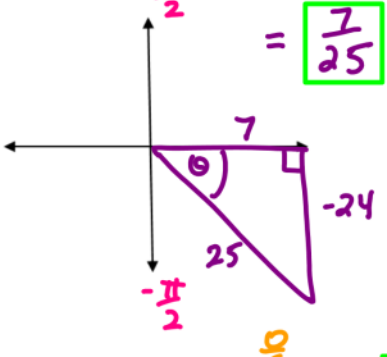


$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

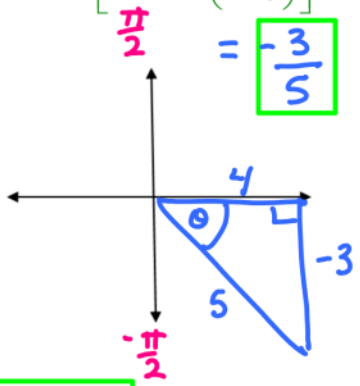
If trig ratios/functions are <u>different</u>, draw a ref. triangle:

Examples:

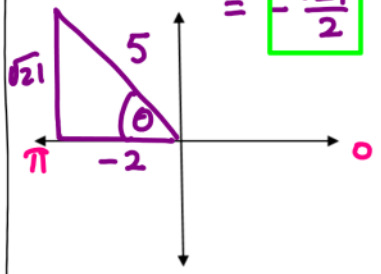
10. $\cos\left[\arcsin\left(-\frac{24}{25}\right)\right] = \boxed{\frac{7}{25}}$



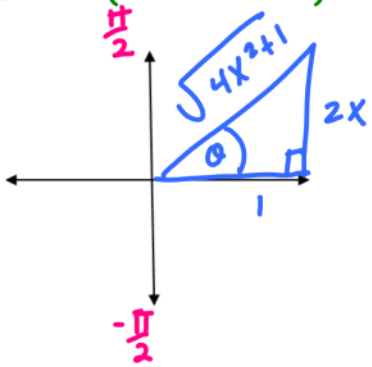
11. $\sin\left[\tan^{-1}\left(-\frac{3}{4}\right)\right] = \boxed{-\frac{3}{5}}$



12. $\tan\left(\arccos -\frac{2}{5}\right) = \boxed{-\frac{\sqrt{21}}{2}}$



7. $\sec(\arctan 2x) = \boxed{\sqrt{4x^2+1}}$



$c^2 = (2x)^2 + 1^2$

$c^2 = 4x^2 + 1$

$c = \sqrt{4x^2+1}$

In assigned groups students work on:

- "Section 4.7 – Inverse Trig Functions"
- Finish for HW