

Sunday, January 06, 2019
4:43 PM

Precalc **KEY**

4.7B: Inverse trig functions

Obj: to evaluate the composition of inverse trig functions & apply inverse properties of trig functions

Hwk: 4.7B #43 - 48 all, 49 - 61 odd

Draw triangles where necessary

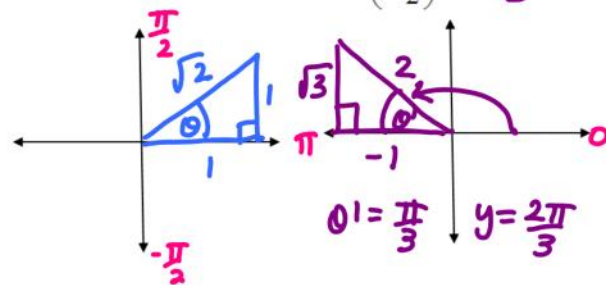
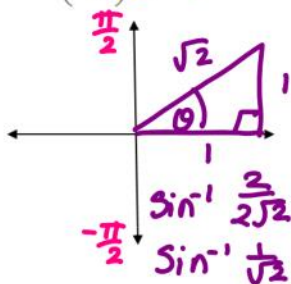
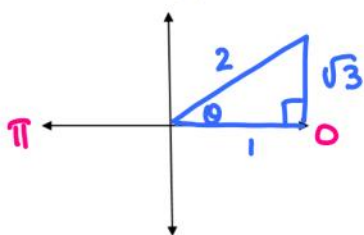
Do Now:

Complete #1 - 8 at the bottom of page.

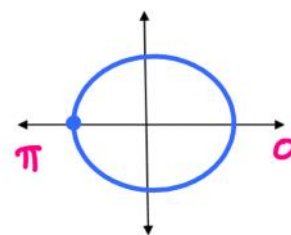
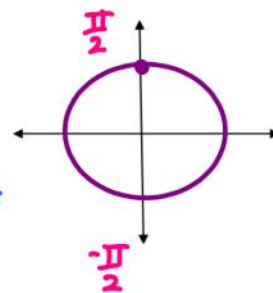
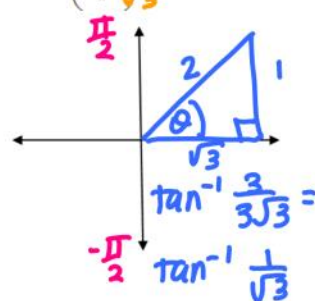
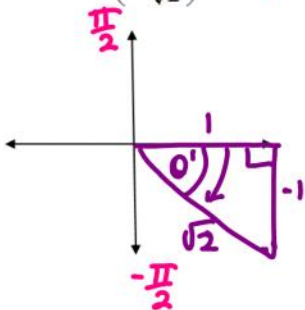
HINT: Draw triangles in appropriate quadrants!

III. (Questions) Find the value of each of the following (without using a calculator):

1. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ 2. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ 3. $\tan^{-1}(1) = \frac{\pi}{4}$ 4. $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



5. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ 6. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ 7. $\sin^{-1}(1) = \frac{\pi}{2}$ 8. $\cos^{-1}(-1) = \pi$



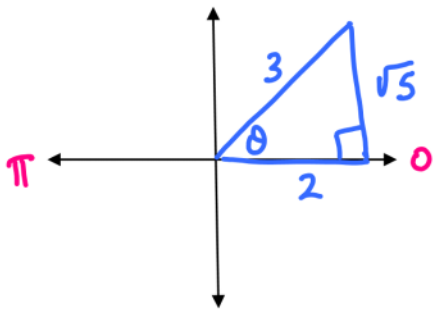
Homework Questions?? 4.7A

Composition of Functions

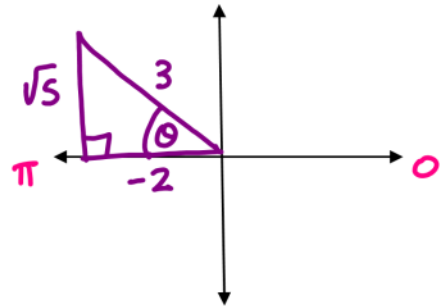
Find the exact value of the expression.

HINT: Sketch a right triangle.

$$\tan \left(\overset{\frac{\theta}{2}}{\text{arccos}} \frac{2}{3} \right) = \boxed{\frac{\sqrt{5}}{2}}$$



$$\sin \left[\overset{\frac{\theta}{2}}{\text{cos}^{-1}} \left(-\frac{2}{3} \right) \right] = \boxed{\frac{\sqrt{5}}{3}}$$



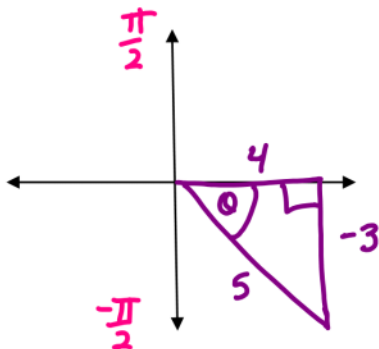
*Remember, when evaluating a trig function, your answer is a ratio.
When evaluating an inverse trig function, your answer is an angle!*

Composition of Functions

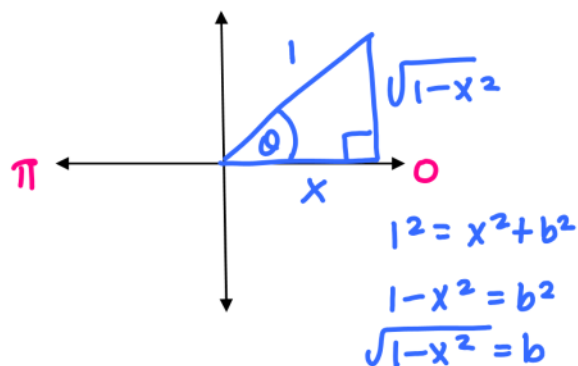
Find the exact value of the expression.

HINT: Sketch a right triangle.

$$\cos \left[\text{arc tan} \left(-\frac{3}{4} \right) \right] = \frac{4}{5}$$



$$\sin \left(\text{arc cos } x \right) = \sqrt{1-x^2}$$



*Remember, when evaluating a trig function, your answer is a ratio.
When evaluating an inverse trig function, your answer is an angle!*

Inverse Properties

Recall, if two functions are inverses,

$$f(g(x)) = g(f(x)) = x$$

or

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x$$

Inverse properties

If $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, then

$$\sin(\arcsin x) = x \text{ and } \arcsin(\sin y) = y$$

If $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$, then

$$\cos(\arccos x) = x \text{ and } \arccos(\cos y) = y$$

If $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$, then

$$\tan(\arctan x) = x \text{ and } \arctan(\tan y) = y$$

Using Inverse Properties

If possible, find the exact value:

Inverse Properties

If $-1 \leq x \leq 1$ and $-\pi/2 \leq y \leq \pi/2$,
then $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$.

If $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$,
then $\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.

If x is a real number and $-\pi/2 < y < \pi/2$,
then $\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.

$$1. \sin\left[\arcsin\left(\frac{2}{3}\right)\right] = \boxed{\frac{2}{3}}$$

$$2. \sin\left[\arcsin\left(\frac{3}{2}\right)\right]$$

* Inverse property doesn't apply.
* $\frac{3}{2}$ is outside of the domain of the arcsine function.

$$3. \cos(\arccos 0.7)$$

$$= \boxed{0.7}$$

$$4. \cos(\cos^{-1} 4)$$

* Inverse property doesn't apply.
* 4 is outside of the domain of the arccos function.

$$5. \tan(\arctan 100)$$

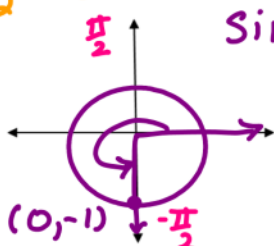
$$= \boxed{100}$$

$$6. \tan^{-1}(\tan \pi) = \boxed{0}$$

* Inverse property doesn't apply.
* π is outside of the range of the arc tan function.

$$7. \arcsin\left[\sin\left(\frac{3\pi}{2}\right)\right] = \boxed{-\frac{\pi}{2}}$$

* Inverse property doesn't apply
* $\frac{3\pi}{2}$ is outside of the range of arcsine.

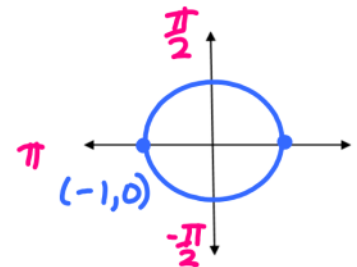


$$\sin \frac{3\pi}{2} = -1$$

$$\arcsin(-1)$$

$$\sin \theta = -1$$

$$\theta = \boxed{-\frac{\pi}{2}} *$$



* So use unit circle!

$$\tan^{-1}(\tan \pi) = \frac{0}{-1}$$

$$\tan^{-1}(0)$$

$$\tan \theta = 0 \quad \theta = \boxed{0}$$

Remember, when evaluating a trig function, your answer is a ratio.

When evaluating an inverse trig function, your answer is an angle!

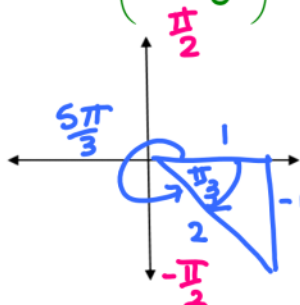
* If the angle is not in the interval, use a co-terminal angle.

Your try.....

If possible, find the exact value:

$$= \arcsin\left(\sin -\frac{\pi}{3}\right)$$

$$8. \arcsin\left(\sin \frac{5\pi}{3}\right) = \boxed{-\frac{\pi}{3}}$$



✗ use a coterminal angle.

because: $\arcsin\left(-\frac{\sqrt{3}}{2}\right) =$
 $\sin \theta = -\frac{\sqrt{3}}{2} \quad \theta = -\frac{\pi}{3}$

Remember, when evaluating a trig function, your answer is a ratio.
 When evaluating an inverse trig function, your answer is an angle!

Inverse Properties

If $-1 \leq x \leq 1$ and $-\pi/2 \leq y \leq \pi/2$,
 then $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$.

If $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$,
 then $\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.

If x is a real number and $-\pi/2 < y < \pi/2$,
 then $\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.

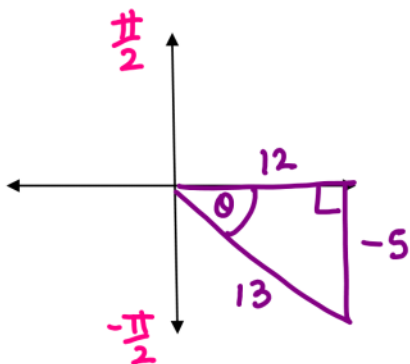
$$9. \tan [\arctan (-14)] = \boxed{-14}$$

Closure

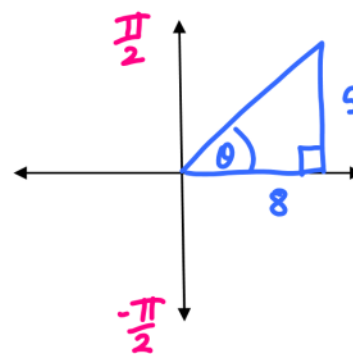
Find the exact value of the expression.

HINT: Sketch a right triangle.

$$\csc \left[\arctan \left(-\frac{5}{12} \right) \right] = \boxed{-\frac{13}{5}}$$

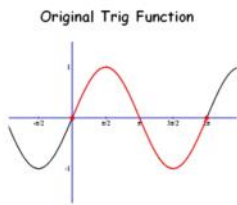


$$\cot \left[\arctan \left(\frac{5}{8} \right) \right] = \boxed{\frac{8}{5}}$$

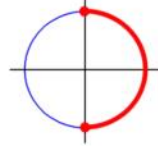


Recap:

$y = \arcsin(x)$ or $\sin^{-1}(x)$

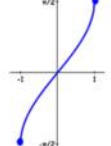


Original Function
Domain (Rest.) & Range

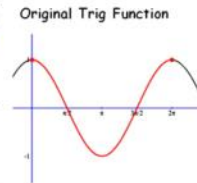


Domain: $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
Range: $-1 \leq \sin\theta \leq 1$

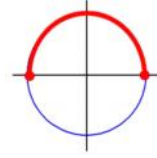
Inverse Function
Domain & Range



Domain: $[-1, 1]$
Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

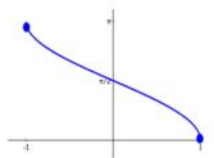


Original Function
Domain (Rest.) & Range



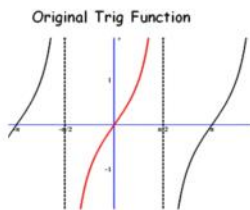
Domain: $0 \leq \theta \leq \pi$
Range: $-1 \leq \cos\theta \leq 1$

Inverse Function
Domain & Range

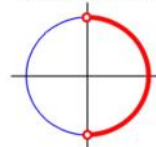


Domain: $[-1, 1]$
Range: $[0, \pi]$

$y = \arctan(x)$ or $\tan^{-1}(x)$

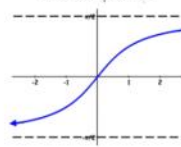


Original Function
Domain (Rest.) & Range

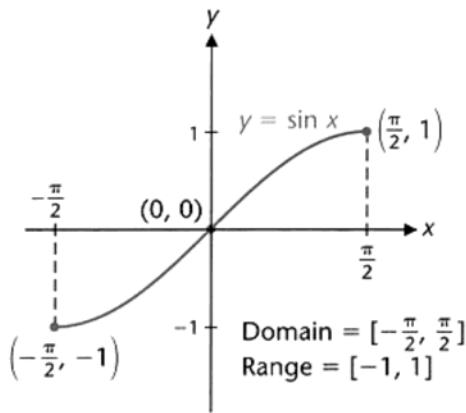


Domain: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
Range: $-\infty < \tan\theta < \infty$

Inverse Function
Domain & Range

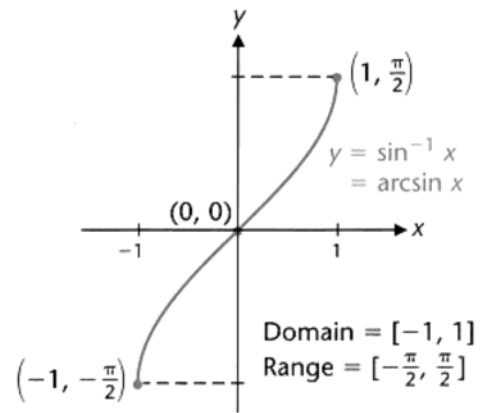


Domain: $(-\infty, \infty)$
Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$



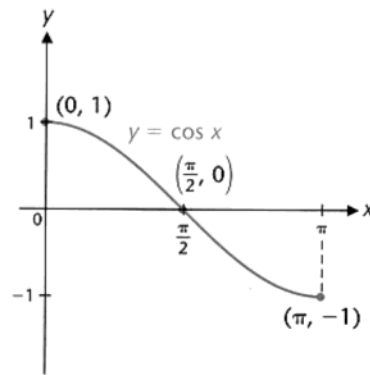
Restricted sine function

(a)



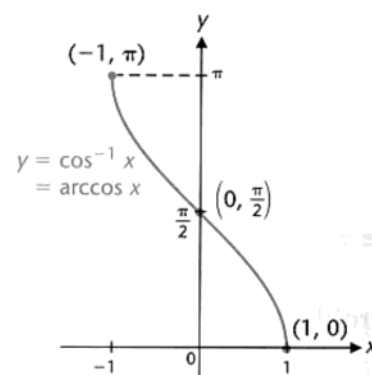
Inverse sine function

(b)



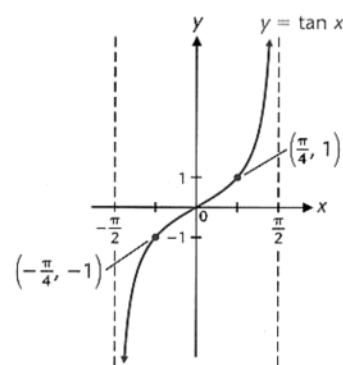
Restricted cosine function

(a)



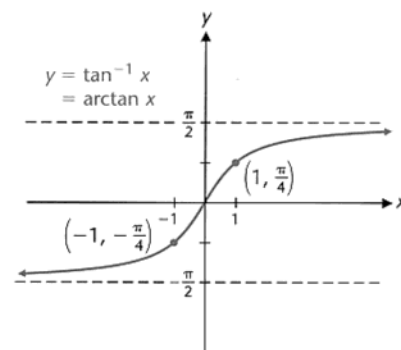
Inverse cosine function

(b)



Restricted tangent function

(a)



Inverse tangent function

(b)