Sunday, January 06, 2019
4:39 PM

Precalc KEY 4.7A: Inverse Trig Functions

Obj: To evaluate inverse trig functions

Hwk: 4.7A #1 – 33 odd (for #1 – 15, you MUST draw triangles)

Do Now:

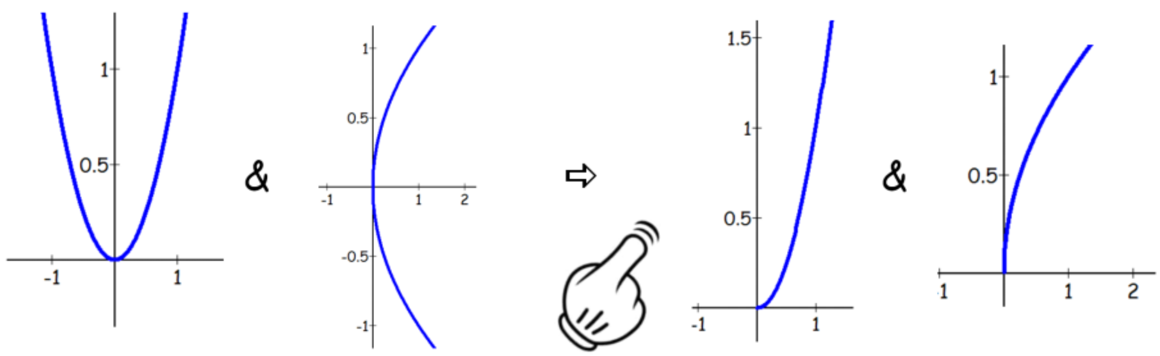
Read & recall:

Inverse ($f^{-1}(x)$) "undoes" operations 

- Swap x and y coordinates
- Domain of original ⇨ range of inverse
- Range of original ⇨ domain of inverse
- To det. if function, use VERTICAL LINE TEST
- To determine if inverse function exists, use HORIZONTAL LINE TEST (HLT)

} One to one function

$f(x) = x^2$ $g(x) = \sqrt{x}$

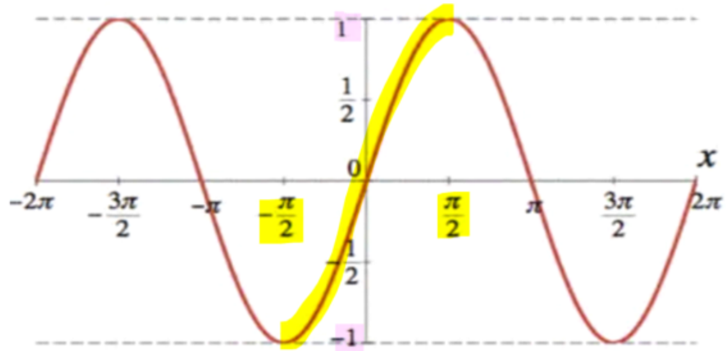


We can RESTRICT THE DOMAIN to ensure an inverse exists.

☆Today we will ***force*** Trig functions to have inverses by ***restricting their domains.***

Inverse Sine Function

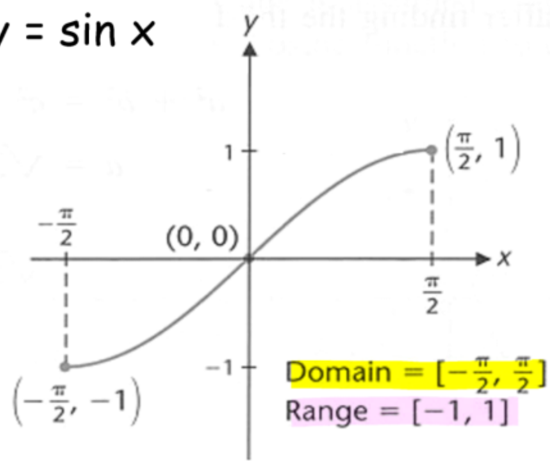
y = sin x does not have an inverse because it is not *one-to-one*.



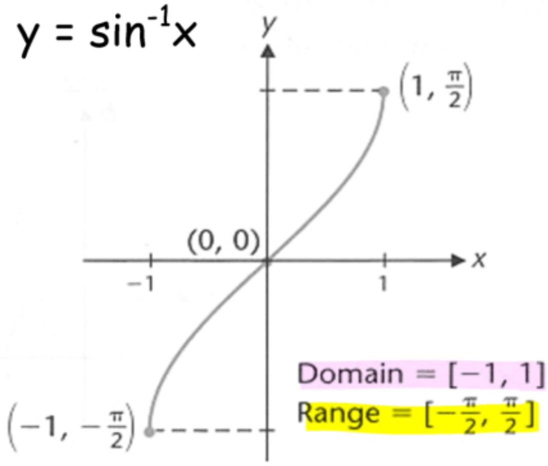
By restricting the domain to [-π/2 ,π/2], the function is now *one-to-one* and passes the *HLT*.

Note that y = sin x takes on the full range of values [-1, 1] in the restricted domain.

y = sin x



y = $\sin^{-1}x$

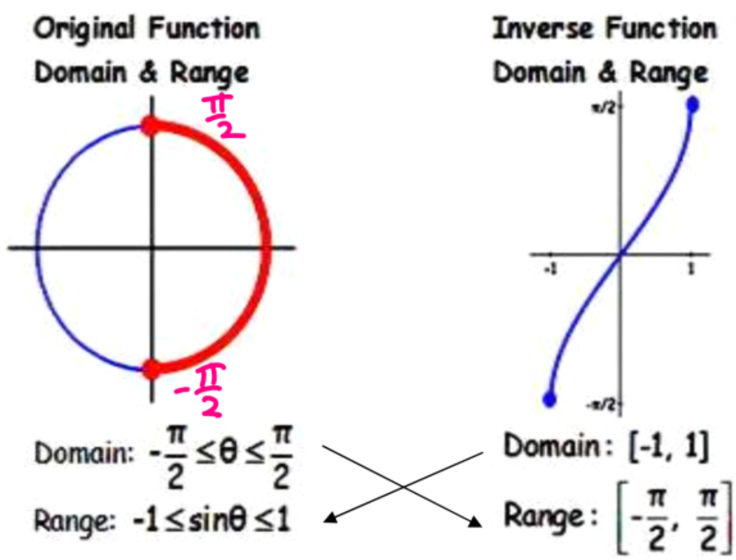


Note: the domain & range are reversed for the inverse function.

Inverse Trig Functions

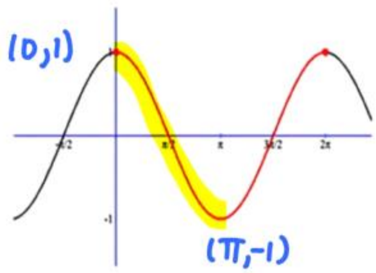
$$y = \sin^{-1} x \qquad \text{or} \qquad y = \text{arc}\sin x$$

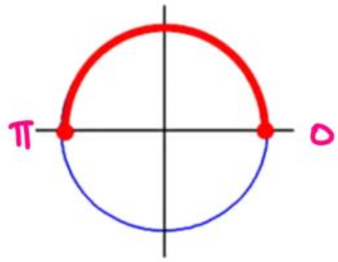
- Read "The angle or arc whose sine is x."
- Another notation for $f^{-1}(x)$
- When evaluating, make sure the argument (x) is within your restricted domain!

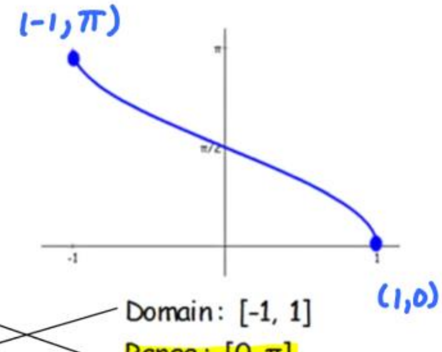


Inverse Trig Functions

$y = \cos^{-1} x$ or $y = \text{arc}\cos x$







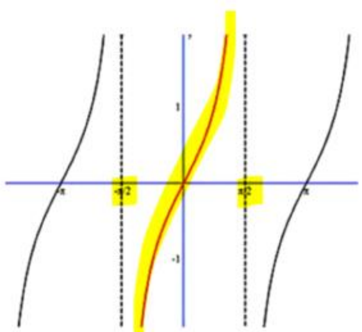
Domain: $0 \le \theta \le \pi$

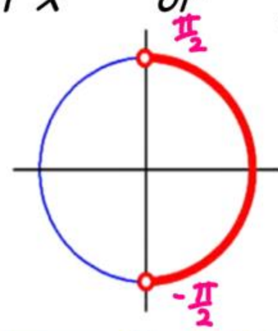
Range: $-1 \le \cos\theta \le 1$

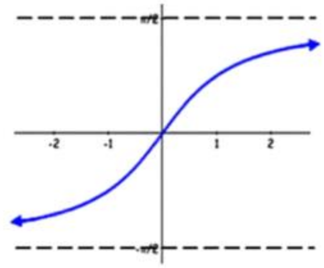
Domain: $[-1, 1]$

Range: $[0, \pi]$

$y = \tan^{-1} x$ or $y = \text{arc}\tan x$







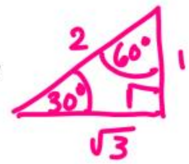
Domain: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Range: $-\infty < \tan\theta < \infty$

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Evaluate the expression without using a calculator

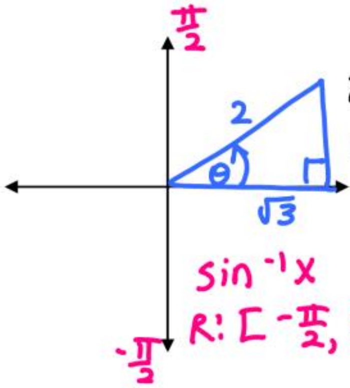


To evaluate, draw a triangle in the correct quadrant.
Then, find the angle with the given trig value (IN RADIANS).
Remember, your domain is restricted!

1. $\sin^{-1}\left(\frac{1}{2}\right)$

$\sin y = \frac{1}{2}$

$y = \boxed{\frac{\pi}{6}}$



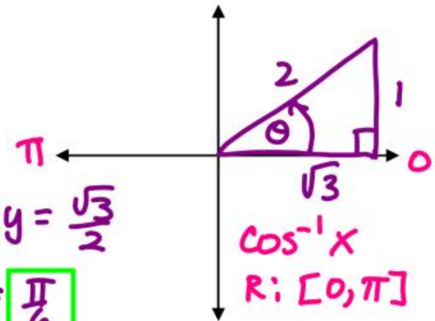
$\sin^{-1} x$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos y = \frac{\sqrt{3}}{2}$

$y = \boxed{\frac{\pi}{6}}$



$\cos^{-1} x$

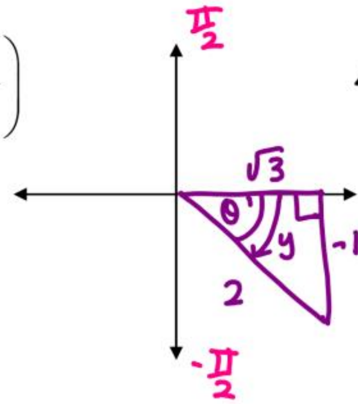
R: $[0, \pi]$

3. $\sin^{-1}\left(-\frac{1}{2}\right)$

$\sin y = -\frac{1}{2}$

$\theta' = \frac{\pi}{6}$

$y = \boxed{-\frac{\pi}{6}}$

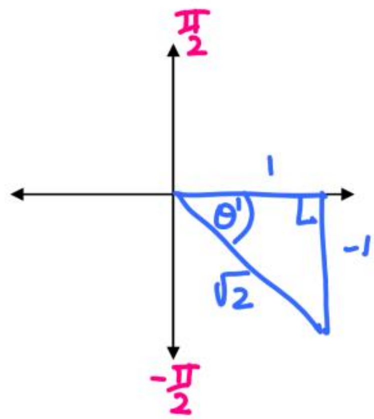


4. arctan(-1)

$\tan\theta = -1$

$\theta' = \frac{\pi}{4}$

$y = \boxed{-\frac{\pi}{4}}$

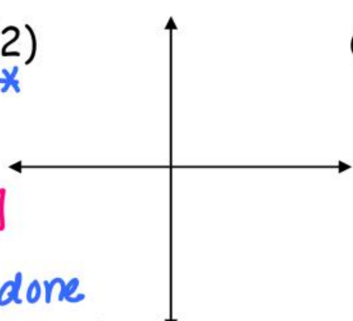


5. arc sin (2) *

arcsin θ

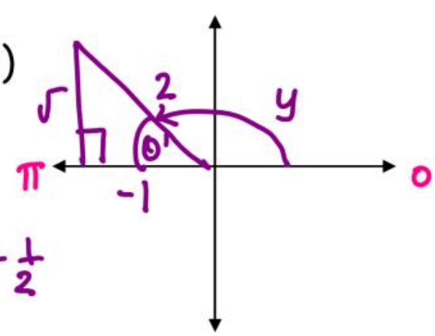
D: $[-1, 1]$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$



* can not be done because 2 is not in the domain.

6. arccos(-.5)



$\cos y = -\frac{1}{2}$

$\theta' = \frac{\pi}{3}$

$y = \boxed{\frac{2\pi}{\pi}}$

Evaluate WITH a calculator. Round to the nearest hundredth.

What MODE should your calculator be in?

7. $\cos^{-1}(0.31)$ = 1.26

8. arctan(13) = 1.49

9. $\sin^{-1}(-.74)$ = -.83

Evaluate the expression without using a calculator.

To evaluate, draw a triangle in the correct quadrant.
Then, find the angle with the given trig value (IN RADIANS).
Remember, your domain is restricted!

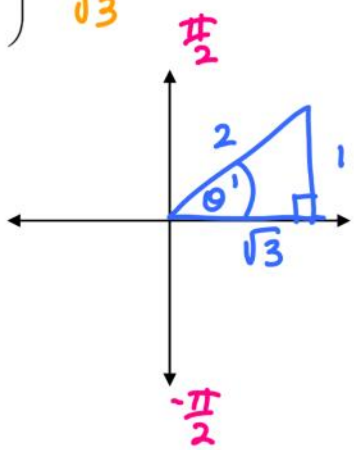
10. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan^{-1} \frac{3}{3\sqrt{3}} =$

$\tan^{-1} \frac{1}{\sqrt{3}}$

$\tan y = \frac{1}{\sqrt{3}}$

$y = \boxed{\frac{\pi}{6}}$



11. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) \cdot \frac{\sqrt{2}}{\sqrt{2}}$

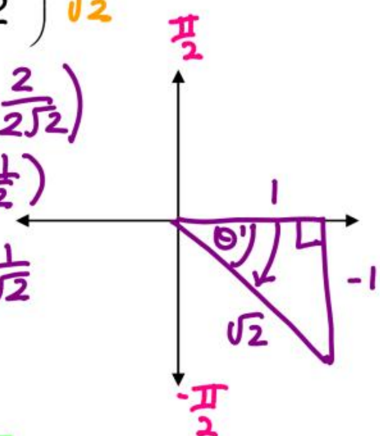
$= \sin^{-1}\left(-\frac{2}{2\sqrt{2}}\right)$

$= \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\sin y = -\frac{1}{\sqrt{2}}$

$\theta' = \frac{\pi}{4}$

$y = \boxed{-\frac{\pi}{4}}$



Closure:

- Why must we restrict the domains of the trig functions to find their inverses? So that the inverses will be functions (pass HLT).
- What are the domain restrictions for sine? R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ D: $[-1, 1]$
- Cosine? R: $[0, \pi]$ D: $[-1, 1]$
- Tangent? R: $(-\frac{\pi}{2}, \frac{\pi}{2})$ D: $(-\infty, \infty)$