

Tuesday, December 12, 2017
6:50 PM

THESE ARE YOUR NOTES!

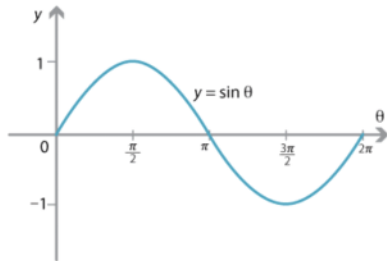
$$y = a \sin(bx - c) + d$$

$$y = a \cos(bx - c) + d$$

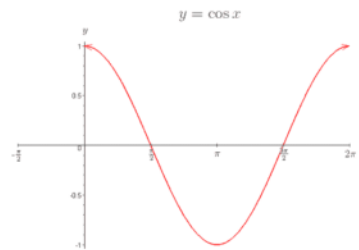
amplitude: $|a|$
 range: $[-a, a] + d$
 period: $\frac{2\pi}{b}$
 start: set $(bx - c) = 0$
 end: set $(bx - c) = 2\pi$
 one cycle: $[start, end]$
 scale: $\frac{period}{4}$

Transformations:
 Vertical stretch $|a| > 1$
 Vertical shrink $|a| < 1$
 Horizontal stretch $|b| < 1$
 Horizontal shrink $|b| > 1$
 Phase shift left $(b(x + c))$
 Phase shift right $(b(x - c))$
 Vertical Shift up $(+d)$
 Vertical Shift down $(-d)$

Graph of Sine



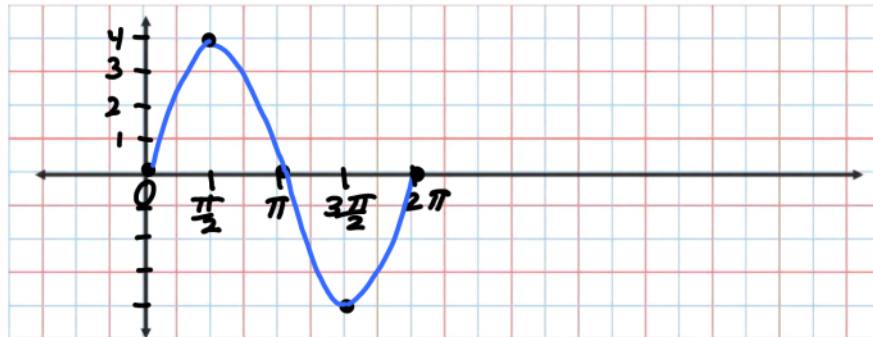
Graph of Cosine



1. Graph $y = 4\sin x$ and identify the following: $a=4$ $b=1$

Amplitude: 4 Range: $[-4, 4]$ Period: 2π Phase Shift: $none$
 * Shift +

* Start: 0 End: 2π One cycle: Scale:
 $bx+c=0$ $x=0$ $bx+c=2\pi$ $[0, 2\pi]$ $period \div 4 = \frac{2\pi}{4} = \frac{\pi}{2}$

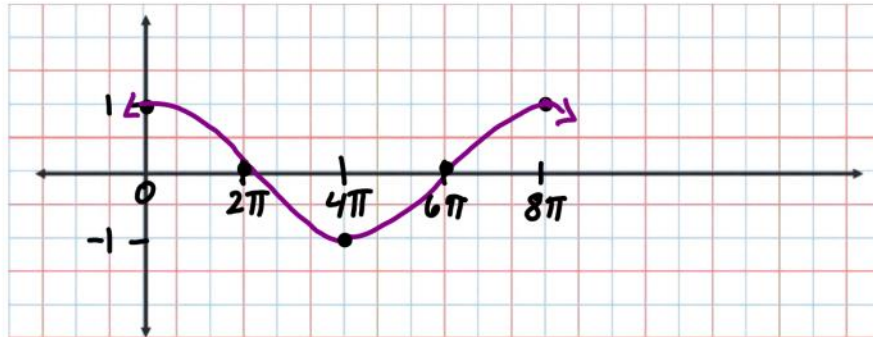


5 Key Points: $(,) (,) (,) (,) (,)$

$(0, 0), (\frac{\pi}{2}, 4), (\pi, 0), (\frac{3\pi}{2}, -4), (2\pi, 0)$

2. Graph $y = \cos\left(\frac{x}{4}\right)$ and identify the following:

Amplitude: 1 Range: $[-1, 1]$ Period: $\frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$ Phase Shift: none
 Start: 0 End: $\frac{x}{4} = (2\pi) \cdot 4$ One cycle: $[0, 8\pi]$ Scale: $\frac{8\pi}{4} = 2\pi$
 $bx - c = 0$ $bx - c = 2\pi$ $x = 8\pi$



5 Key Points: (,) (,) (,) (,) (,)

$(0, 1), (2\pi, 0), (4\pi, -1), (6\pi, 0), (8\pi, 1)$

3. Graph $y = -\frac{1}{2}\sin\left(\frac{x}{3}\right) + 2$ and identify the following: $a = -\frac{1}{2}$ $b = \frac{1}{3}$ $d = 2$

Amplitude: $|-a| = \frac{1}{2}$ Range: $[1.5, 2.5]$ Period: $\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$ Phase Shift: none
 Start: 0 End: 6π One cycle: $[0, 6\pi]$ Scale: $\frac{6\pi}{4} = \frac{3\pi}{2}$
 $bx - c = 0$ $\frac{x}{3} = 0$ $x = 0$ $bx - c = 2\pi$ $\frac{x}{3} = 2\pi$ $x = 6\pi$



5 Key Points: (,) (,) (,) (,) (,)

$(0, 2), (3\pi/2, 1.5), (3\pi, 2), (9\pi/2, 2.5), (6\pi, 2)$

$$\cos\left(\frac{2\pi}{3}x\right) + 5 \quad a=1 \quad b=\frac{2\pi}{3} \quad d=5$$

4. Graph $y = 5 + \cos\left(\frac{2\pi}{3}x\right)$ and identify the following:

Amplitude: 1

Range: $[4, 6]$

$$\frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{3}} = 3$$

Phase Shift: none

Start: 0

End: 3

One cycle: $[0, 3]$

Scale:

$$bx - c = 0 \\ \frac{2\pi}{3}x = 0$$

$$bx - c = 2\pi \\ \frac{2\pi}{3}x = 2\pi \quad x = 2\pi \cdot \frac{3}{2\pi}$$

$$\text{period} \div 4 = \frac{3}{4}$$



5 Key Points: (,) (,) (,) (,) (,)

$$(0, 6), \left(\frac{3}{4}, 5\right), \left(\frac{3}{2}, 4\right), \left(\frac{9}{4}, 5\right), (3, 6)$$

5. Graph $y = 2\sin\left(x - \frac{\pi}{2}\right)$ and identify the following: $a=2 \quad b=1 \quad c=\frac{\pi}{2} \quad d=0$

Amplitude: 2

Range: $[-2, 2]$

Period: 2π

Phase Shift: $\frac{\pi}{2}$ Right

Start: $\frac{\pi}{2}$

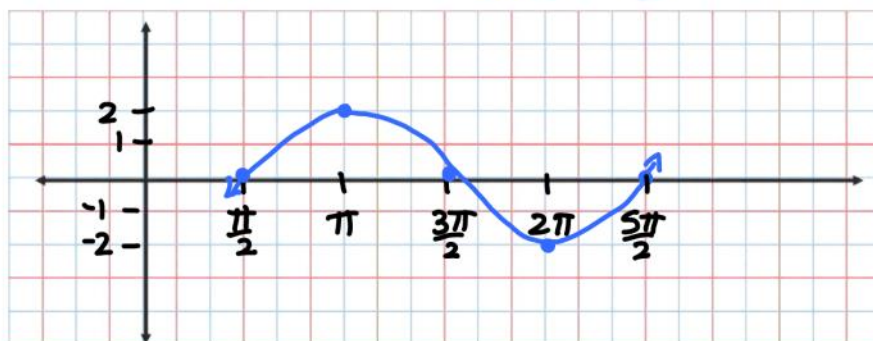
End: $\frac{5\pi}{2}$

One cycle: $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$

Scale: $\frac{2\pi}{4} = \frac{\pi}{2}$

$$bx - c = 0 \\ x - \frac{\pi}{2} = 0 \\ x = \frac{\pi}{2}$$

$$bx - c = 2\pi \\ x - \frac{\pi}{2} = 2\pi \quad x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$



5 Key Points: (,) (,) (,) (,) (,)

$$\left(\frac{\pi}{2}, 0\right), (\pi, 2), \left(\frac{3\pi}{2}, 0\right), (2\pi, -2), \left(\frac{5\pi}{2}, 0\right)$$

6. Graph $y = -\cos(2\pi x + 4\pi)$ and identify the following: $a = -1$ $b = 2\pi$ $c = 4\pi$ $d = 0$

Amplitude: $|-1| = 1$ Range: $[-1, 1]$ Period: $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$ Phase Shift: 2 left

Start: -2

End: -1

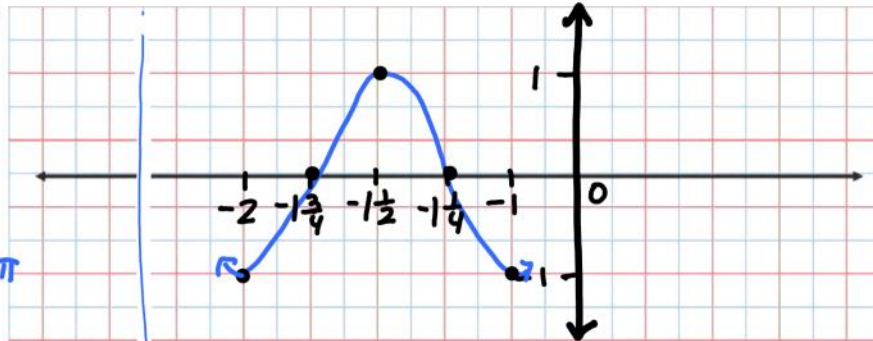
One cycle: $[-2, -1]$

Scale: $\text{Period} \div 4$
 $\frac{1}{4}$

$$\begin{aligned} bx - c &= 0 \\ 2\pi x + 4\pi &= 0 \\ 2\pi x &= -4\pi \\ x &= \frac{-4\pi}{2\pi} \\ x &= -2 \end{aligned}$$

$$\begin{aligned} bx - c &= 2\pi \\ * \text{ see below} \end{aligned}$$

$$\begin{aligned} * bx - c &= 2\pi \\ 2\pi x + 4\pi &= 2\pi \\ 2\pi x &= -2\pi \\ x &= -1 \end{aligned}$$



5 Key Points: (,) (,) (,) (,) (,)

$(-2, -1), (-1\frac{3}{4}, 0), (-1\frac{1}{2}, 1), (-1\frac{1}{4}, 0), (-1, -1)$

7. Graph $y = \sin(\frac{x}{2})$ and identify the following: $a = 1$ $b = \frac{1}{2}$

* Reflect over X-axis

Amplitude: $|-1| = 1$ Range: $[-1, 1]$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Phase Shift: none

Start: 0

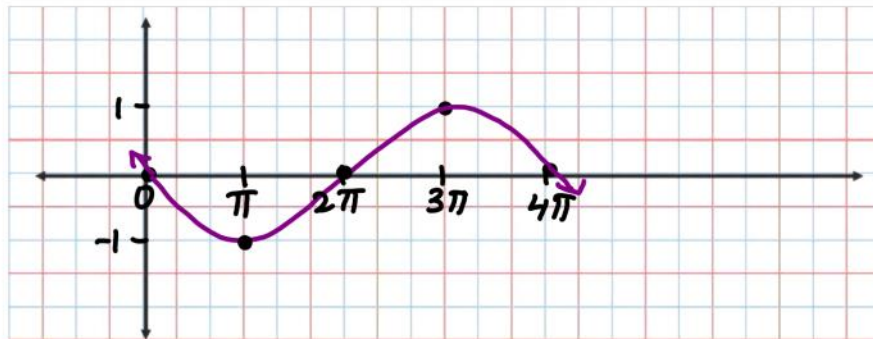
End: 4π

One cycle: $[0, 4\pi]$

Scale: $\text{Period} \div 4$
 $\frac{4\pi}{4} = \pi$

$$\begin{aligned} bx - c &= 0 \\ \frac{x}{2} &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} bx - c &= 2\pi \\ 2(\frac{x}{2}) &= (2\pi) \cdot 2 \\ x &= 4\pi \end{aligned}$$



5 Key Points: (,) (,) (,) (,) (,)

$(0, 0), (\pi, -1), (2\pi, 0), (3\pi, 1), (4\pi, 0)$

$$[-a, a] + d$$

8. Graph $y = 3 \cos\left(\frac{x}{4} - \frac{\pi}{2}\right) + 2$ and identify the following:

$$a = 3 \quad b = \frac{1}{4} \quad c = \frac{\pi}{2} \quad d = 2$$

Amplitude: 3

Range: $[-1, 5]$

Period:

$$\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$$

Phase Shift:

2π Right

Start: 2π

End: 10π

One cycle:

$$[2\pi, 10\pi)$$

Scale:

period $\div 4$

$$\frac{8\pi}{4} = 2\pi$$

$$bx - c = 0$$

$$\frac{x}{4} - \frac{\pi}{2} = 0$$

$$4\left(\frac{x}{4}\right) = \left(\frac{\pi}{2}\right) \cdot 4$$

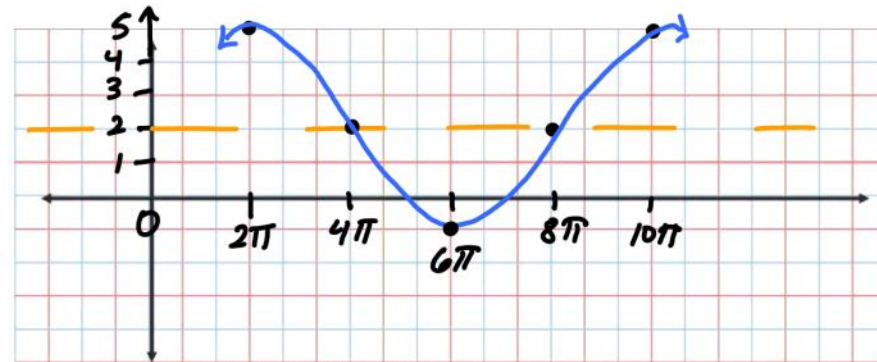
$$x = 2\pi$$

$$bx - c = 2\pi$$

$$\frac{x}{4} - \frac{\pi}{2} = 2\pi$$

$$\frac{x}{4} = 2\pi + \frac{\pi}{2} + \frac{\pi}{2}$$

$$4\left(\frac{x}{4}\right) = \left(\frac{5\pi}{2}\right) \cdot 4$$



Line of oscillation $y = 2$

5 Key Points: (,) (,) (,) (,) (,)

$$(2\pi, 5), (4\pi, 2), (6\pi, -1), (8\pi, 2), (10\pi, 5)$$

9. Graph $y = 5 \sin(x + \pi) - 3$ and identify the following:

$$a = 5 \quad b = 1 \quad c = -\pi \quad d = -3$$

Amplitude: 5

Range: $[-8, 2]$

Period:

$$\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$

Phase Shift:

π left

Start: $-\pi$

End: π

One cycle:

$$[-\pi, \pi]$$

Scale:

period $\div 4$

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

$$bx - c = 0$$

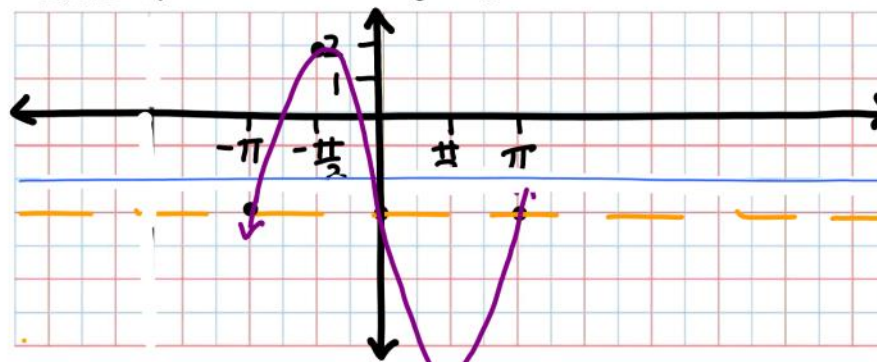
$$x + \pi = 0$$

$$x = -\pi$$

$$bx - c = 2\pi$$

$$x + \pi = 2\pi$$

$$x = \pi$$



Line of oscillation $y = -3$

5 Key Points: (,) (,) (,) (,) (,)

$$(-\pi, -3), \left(-\frac{\pi}{2}, 2\right), (0, -3), \left(\frac{\pi}{2}, -8\right), (\pi, -3)$$

$$y = 2\cos\left(\frac{1}{2}x\right) - 3 \quad a=2 \quad b=\frac{1}{2} \quad d=-3$$

10. Graph $y = -3 + 2\cos\left(\frac{x}{2}\right)$ and identify the following:

Amplitude: 2

Range: $[-5, -1]$

Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: none

Start: 0

End: 4π

One cycle: $[0, 4\pi]$

Scale: $\text{Period} \div 4$
 $\frac{4\pi}{4} = \pi$

$$bx - c = 0$$

$$\frac{x}{2} = 0$$

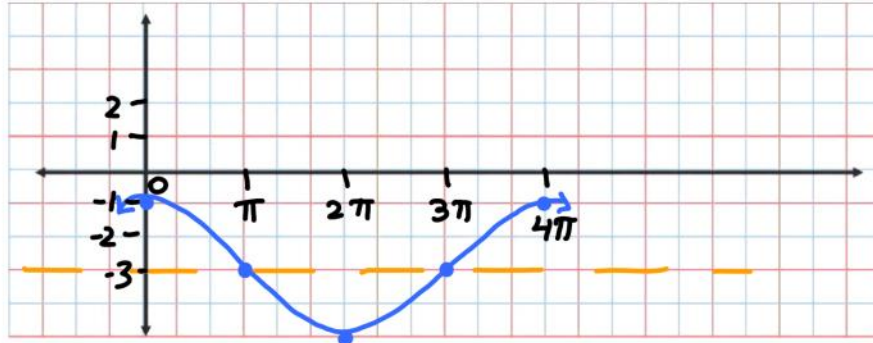
$$x = 0$$

$$bx - c = 2\pi$$

$$\frac{1}{2}x = 2\pi$$

$$x = 4\pi$$

Line of Oscillation: $y = -3$



5 Key Points: (,) (,) (,) (,) (,)

$$(0, -1), (\pi, -3), (2\pi, -1), (3\pi, -3), (4\pi, -1)$$

11. Graph $y = \frac{1}{2}\cos\left(x - \frac{\pi}{3}\right)$ and identify the following:

$$a = \frac{1}{2} \quad b = 1 \quad c = \frac{\pi}{3}$$

Amplitude: $\frac{1}{2}$

Range: $[-\frac{1}{2}, \frac{1}{2}]$

Period: $\frac{2\pi}{1} = 2\pi$

Phase Shift: $\frac{\pi}{3}$ right
Scale:

Start: $\frac{\pi}{3} \cdot \frac{2}{2} = \frac{2\pi}{6}$

End: $\frac{7\pi}{3} \cdot \frac{2}{2} = \frac{14\pi}{6}$

One cycle: $[\frac{\pi}{3}, \frac{7\pi}{3}]$

Period $\div 4$
 $\frac{2\pi}{4} = \frac{\pi}{2}$
 $\frac{\pi}{2} \cdot \frac{2}{3} = \frac{3\pi}{6}$

$$bx - c = 0$$

$$x - \frac{\pi}{3} = 0$$

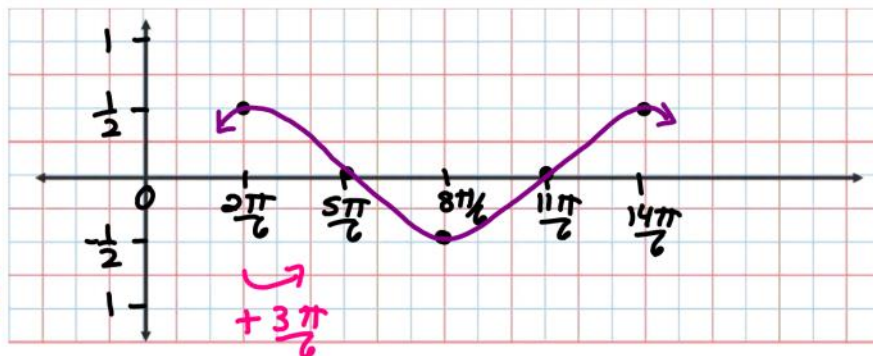
$$x = \frac{\pi}{3}$$

$$bx - c = 2\pi$$

$$x - \frac{\pi}{3} = 2\pi$$

$$x = 2\pi + \frac{\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3} = \frac{7\pi}{3}$$

$$x = \frac{7\pi}{3}$$



5 Key Points: (,) (,) (,) (,) (,)

$$\left(\frac{\pi}{3}, \frac{1}{2}\right), \left(\frac{5\pi}{6}, 0\right), \left(\frac{4\pi}{3}, -\frac{1}{2}\right), \left(\frac{11\pi}{6}, 0\right), \left(\frac{7\pi}{3}, \frac{1}{2}\right)$$