Thursday, December 06, 2018
7:17 PM

KEY

Precalc 4.5A: Graphs of Sin & Cos Functions

Obj: To graph sine and cosine functions and identify the period and amplitude of each

Hwk: 4.5A #1 – 14 all

4.5 – 4.6 Quiz - Weds 12/19

Do Now:

- Fill in each chart for sin θ and cos θ using the given Unit Circle.
  ☆ Remember, sin θ = y and cos θ = x.
- We will plot the points together.

Think back to Algebra –

How did we first start graphing linear, quadratic, cubic, and absolute value functions?

First we made a chart and then graphed! Then we looked for Patterns and wrote equations to represent them.

Recap:

$$y = a(x - b)^2 + c$$

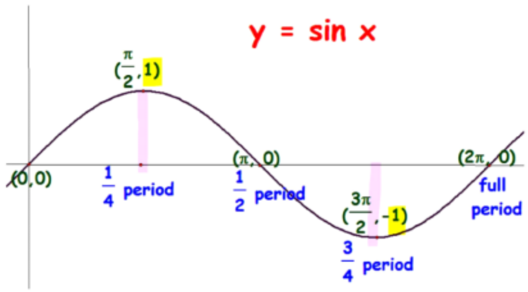
- vertical shift

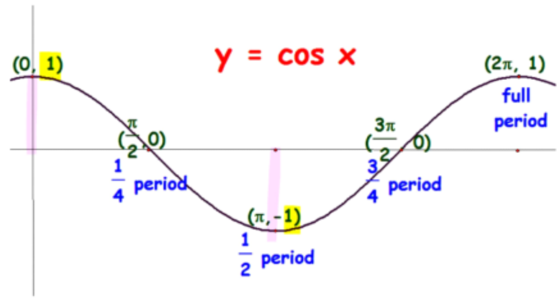
- opens up/down
- vertical stretch/shrink

- horizontal shift
- do "opposite"

Today we are graphing ***sine*** and ***cosine*** functions.

Sine & Cosine Graphs





The sine curve is symmetric about origin → ODD
The cosine curve is symmetric about y-axis → EVEN

Domain: (x-values): all real numbers or (-∞, ∞)
Range: (y-values): [-1, 1]

The period is one ***cycle*** of the graph or the interval in which all range of values – from min. to max. appear.

The graphs repeat indefinitely in BOTH directions
The patterns continue periodically.

Sine & Cosine Graphs

$y = a \sin(bx - c) + d$ $\qquad$ $y = a \cos(bx - c) + d$

Amplitude: $|a|$

- Half the distance between the min. & max. values
- Vertical stretch/shrink
- If $|a| < 1 \rightarrow$ vertical shrink, If $|a| > 1 \rightarrow$ vertical stretch
- If $a < 0$, reflect over the x-axis

Period: $\frac{2\pi}{b}$

- 1 cycle of graph
- Interval in which all range of values (min to max) appear.

Line of Oscillation:

- $y = d$
- The line that divides the graph vertically into 2 equal parts.
- "center line"

$y = a\sin(bx - c) + d \qquad y = a\cos(bx - c) + d$

Example 1:

Identify the period and amplitude of each function. Then define the range, beginning and end of one cycle.

set $bx - c = 0$

set $bx - c = 2\pi$

$a = 4 \quad b = 3$

a. $y = 4\cos 3x$

$\frac{2\pi}{b}$ Per: $\boxed{\frac{2\pi}{3}}$

Amp: $|4| = \boxed{4}$

Range: $\boxed{[-4, 4]}$

$bx - c = 0$ Beg: $\frac{3x}{3} = \frac{0}{3} \quad x = \boxed{0}$

$bx - c = 2\pi$ End: $3x = 2\pi \quad x = \boxed{\frac{2\pi}{3}}$

$a = -\frac{3}{4} \quad b = \frac{1}{2}$

b. $y = -\frac{3}{4}\sin\left(\frac{1}{2}x\right)$

Per: $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = \boxed{4\pi}$

Amp: $\left|-\frac{3}{4}\right| = \boxed{\frac{3}{4}}$

Range: $\boxed{\left[-\frac{3}{4}, \frac{3}{4}\right]}$

Beg: $2\left(\frac{1}{2}x\right) = (0)2 \quad x = \boxed{0}$

End: $2\left(\frac{1}{2}x\right) = (2\pi)2 \quad x = \boxed{4\pi}$

c. $y = 2 + \cos 2x$ $\quad a = 1 \quad b = 2 \quad d = 2$

$y = \cos 2x + 2$

$\frac{2\pi}{b}$ Per: $\frac{2\pi}{2} = \boxed{\pi}$

Amp: $|1| = \boxed{1}$

Range: $\boxed{[-1, 1]}$

$bx + c = 0$ Beg: $\frac{2x}{2} = \frac{0}{2} \quad x = \boxed{0}$

$bx + c = 2\pi$ End: $\frac{2x}{2} = \frac{2\pi}{2} \quad x = \boxed{\pi}$

d. $y = \frac{1}{4}\cos\frac{\pi x}{2}$ $\quad a = \frac{1}{4} \quad b = \frac{\pi}{2}$

$\frac{2\pi}{b}$ Per: $\frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = \boxed{4}$

Amp: $\boxed{\frac{1}{4}}$

Range: $\boxed{\left[-\frac{1}{4}, \frac{1}{4}\right]}$

Beg: $\left(\frac{2}{\pi}\right)\frac{\pi x}{2} = (0)\frac{2}{\pi} \quad x = \boxed{0}$

End: $\frac{2}{\pi}\left(\frac{\pi x}{2}\right) = (2\pi)\frac{2}{\pi} = \boxed{4}$

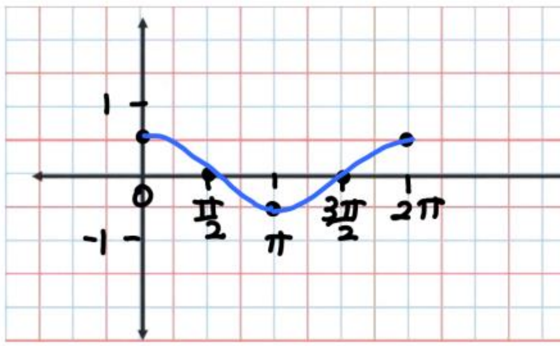
Example 2:

Identify the period & amplitude of each function.
Then define the range and interval for one cycle.

Hint: To determine where one cycle will begin and end,
set ***bx-c =0*** *and* ***bx-c = 2*** *π.*

$a = \frac{1}{2}$ $b = 1$

a) $y = \frac{1}{2}\cos x$



1a) Amplitude: $\frac{1}{2}$

Period: $\frac{2\pi}{1} = 2\pi$

Scale: period $\div$ 4

$\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

Start: 0

End: 2π

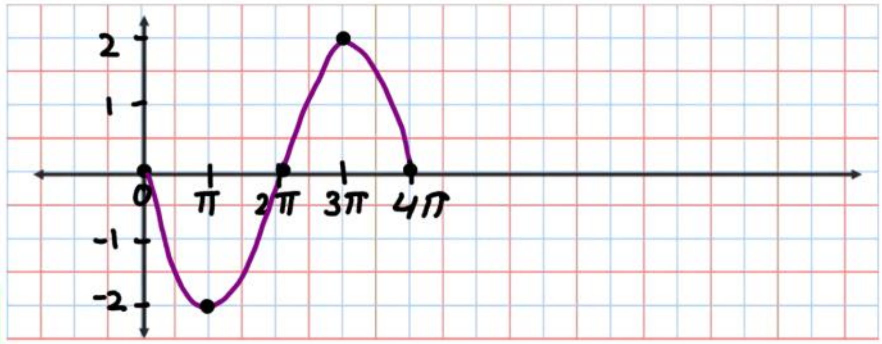
5 key points: $(0, \frac{1}{2})$, $(\frac{\pi}{2}, 0)$, $(\pi, -\frac{1}{2})$, $(\frac{3\pi}{2}, 0)$, $(2\pi, \frac{1}{2})$

Think about g(x) = ***a****f(x). How does* ***a*** *affect the graph?*

$a = -2 \quad b = \frac{1}{2}$

b) $y = -2\sin\left(\frac{1}{2}x\right)$

* Reflect over x-Axis!



|a| Amplitude: 2

$\frac{2\pi}{b}$ Period: $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = \boxed{4\pi}$

Scale:
period ÷ 4 $\quad \frac{4\pi}{4} = \boxed{\pi}$

Start: $bx - c = 0 \quad \frac{1}{2}x = 0 \quad x = \boxed{0}$

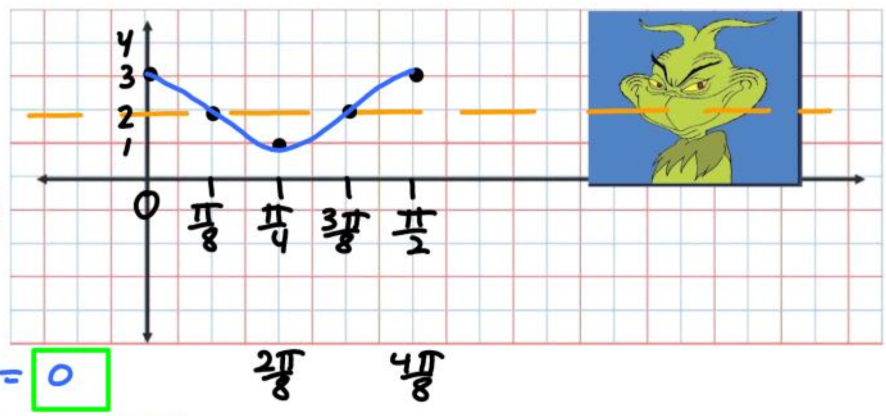
End: $bx - c = 2\pi \quad 2\left(\frac{1}{2}x\right) = (2\pi)2 \quad x = \boxed{4\pi}$

5 key points: $(0,0), (\pi,-2), (2\pi,0), (3\pi,2), (4\pi,0)$

$y = \cos(4x) + 2 \quad a = 1 \quad b = 4 \quad c = 0 \quad d = 2$

* Shift up 2

c) $y = 2 + \cos(4x)$



|a| Amplitude: 1

$\frac{2\pi}{b}$ Period: $\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

Scale:
Period ÷ 4 $\quad \frac{\frac{\pi}{2}}{4} = \frac{\pi}{2} \cdot \frac{1}{4} = \boxed{\frac{\pi}{8}}$

Start:
$bx - c = 0 \quad 4x = 0 \quad x = \boxed{0}$

End:
$bx - c = 2\pi \quad \frac{4x}{4} = \frac{2\pi}{4} \quad x = \boxed{\frac{\pi}{2}}$

5 key points:

Line of oscillation: $y = d \quad \boxed{y = 2}$

$a=4 \quad b=\frac{\pi}{2} \quad c=0 \quad d=-1$

d) $y = 4\sin\left(\frac{\pi}{2}x\right) - 1$

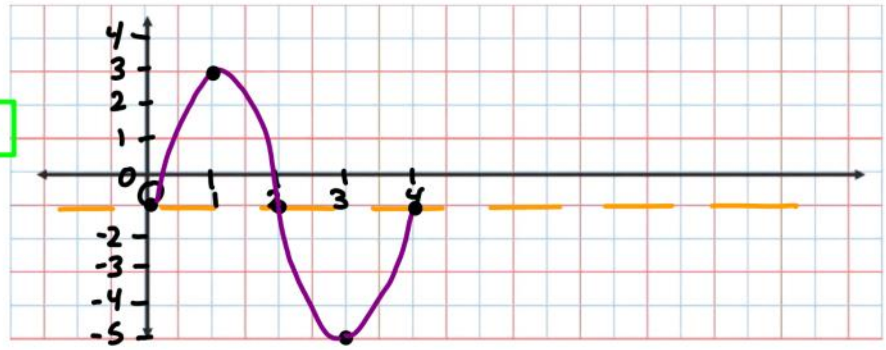
Amplitude: 4

$\frac{2\pi}{b}$ Period:

$\frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = \boxed{4}$

Scale:

period ÷ 4 $\quad \frac{4}{4} = \boxed{1}$



* shifted down 1

Start: $\boxed{0}$

$bx-c=0$

End: $\frac{2}{\pi}\left(\frac{\pi}{2}\right)x = (2\pi)\frac{2}{\pi} \qquad x = \boxed{4}$

$bx-c=2\pi$

5 key points:

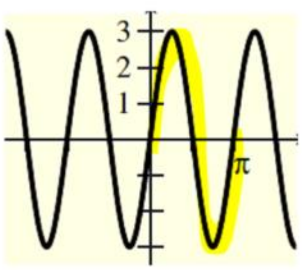
(0,−1), (1,3), (2,−1), (3,−5), (4,−1)

Example 3:

Identify the period & amplitude of the function. Then define the range and interval for one cycle.

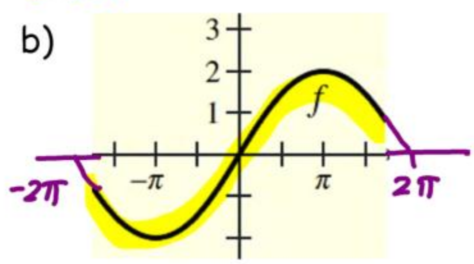
START & END

a)



Period = π
amplitude = 3
range = [−3, 3]
Interval: 0 to π

b)



Period = 4π
Amplitude = 2
Range: [−2, 2]
Interval: −2π to 2π

Closure:

*Think about g(x) = **a**f(**b**x) + **d**. How do a, b & d affect the graph?*

$$y = 2 + 4\cos(3x)$$

$y = 4\cos(3x+2) + 2$

$a = 4$ vertical stretch

$d = 2$ shift up 2

$bx + c = 0$ $3x = 0$

$x = 0$

$bx + c = 2\pi$ $3x = 2\pi$

$x = \frac{2\pi}{3}$

0 to $\frac{2\pi}{3}$

$b = 3$ HORIZontal compress