

Saturday, November 25, 2017
4:14 PM

4.2 In Exercises 25–28, find the point (x, y) on the unit circle that corresponds to the real number t .

$$25) t = \frac{2\pi}{3}$$

$$\text{point: } \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$27) t = \frac{5\pi}{6}$$

$$\text{point: } \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

In Exercises 29–32, evaluate (if possible) the six trigonometric functions of the real number.

$$\begin{array}{ll} \sin t = y & \csc t = \frac{1}{y} \\ \cos t = x & \sec t = \frac{1}{x} \\ \tan t = \frac{y}{x} & \cot t = \frac{x}{y} \end{array}$$

$$29) t = \frac{7\pi}{6} \quad \text{point: } \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\sin(t) = y = \boxed{-\frac{1}{2}}$$

$$\csc(t) = \boxed{-2}$$

$$\cos(t) = x = \boxed{-\frac{\sqrt{3}}{2}}$$

$$\sec(t) = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$$

$$\tan(t) = \frac{y}{x} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$\cot(t) = \boxed{\sqrt{3}}$$

$$= -\frac{1}{2} \cdot \left(-\frac{2}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

$$31) t = -\frac{2\pi}{3} \quad \text{point: } \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\sin(t) = y = \boxed{-\frac{\sqrt{3}}{2}}$$

$$\csc(t) = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$$

$$\cos(t) = x = \boxed{-\frac{1}{2}}$$

$$\sec(t) = \boxed{-2}$$

$$\tan(t) = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$\cot(t) = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

$$= -\frac{\sqrt{3}}{2} \cdot \left(-\frac{2}{1}\right) = \boxed{\sqrt{3}}$$

In Exercises 37–40, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places.

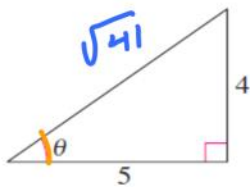
$$37) \tan 33 = \boxed{-75.3130}$$

*radian mode!
(no ° symbol)

$$39) \sec \frac{12\pi}{5} = \frac{1}{\cos \frac{12\pi}{5}} = \boxed{3.2361}$$

4.3 In Exercises 41 and 43 find the exact values of the six trigonometric functions of the angle θ shown in the figure.

41.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + 5^2 &= c^2 \\ 16 + 25 &= c^2 \\ 41 &= c^2 \\ c &= \sqrt{41} \end{aligned}$$

$$\frac{o}{h} \sin \theta = \frac{4}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \boxed{\frac{4\sqrt{41}}{41}}$$

$$\csc \theta = \boxed{\frac{\sqrt{41}}{4}}$$

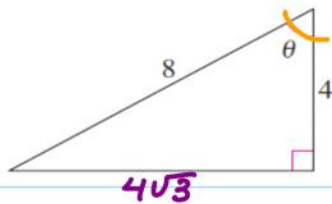
$$\frac{a}{h} \cos \theta = \frac{5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \boxed{\frac{5\sqrt{41}}{41}}$$

$$\sec \theta = \boxed{\frac{\sqrt{41}}{5}}$$

$$\frac{o}{a} \tan \theta = \boxed{\frac{4}{5}}$$

$$\cot \theta = \boxed{\frac{5}{4}}$$

43.



$$\begin{aligned} a^2 + 4^2 &= 8^2 \\ a^2 + 16 &= 64 \\ a^2 &= 48 \\ a &= \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3} \end{aligned}$$

$$\frac{o}{h} \sin \theta = \frac{4\sqrt{3}}{8} = \boxed{\frac{\sqrt{3}}{2}}$$

$$\csc \theta = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$$

$$\frac{a}{h} \cos \theta = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$$\sec \theta = \boxed{2}$$

$$\frac{o}{a} \tan \theta = \frac{4\sqrt{3}}{4} = \boxed{\sqrt{3}}$$

$$\cot \theta = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

In Exercises 45 - 48 use the given function value and **trigonometric identities** (including the cofunction identities) to find the indicated trigonometric functions.

* Pythagorean Identity
 $\sin^2 \theta + \cos^2 \theta = 1$

47. $\csc \theta = 4$ (a) $\sin \theta$ (b) $\cos \theta$
 (c) $\sec \theta$ (d) $\tan \theta$

$(\frac{1}{4})^2 + \cos^2 \theta = 1$
 $\frac{1}{16} + \cos^2 \theta = 1$
 $\cos^2 \theta = \frac{15}{16}$
 $\cos \theta = \sqrt{\frac{15}{16}}$

a) $\sin \theta = \frac{1}{\csc \theta} = \frac{1}{4}$

b) $\cos \theta = \frac{\sqrt{15}}{4}$

c) $\sec \theta = \frac{1}{\cos \theta} = \frac{4}{\frac{\sqrt{15}}{4}} = \frac{4 \cdot 4}{\sqrt{15}} = \frac{16}{\sqrt{15}}$

d) $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{4}}{\frac{\sqrt{15}}{4}} = \frac{1}{4} \cdot \frac{4}{\sqrt{15}} = \frac{1}{\sqrt{15}}$

In Exercises 49 - 54, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places.

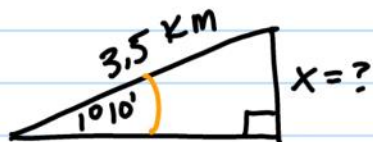
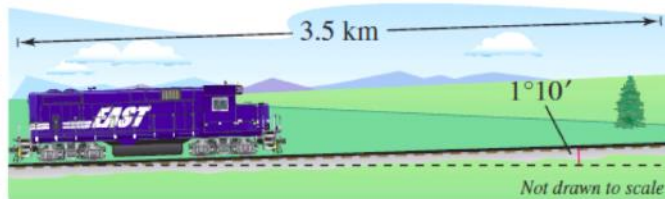
* Degree mode!

49) $\tan 33^\circ = .6494$

51) $\sin 34.2^\circ = .5621$

53) $\cot 15^\circ 14' = \frac{1}{\tan 15^\circ 14'} = 3.6722$

55. **RAILROAD GRADE** A train travels 3.5 kilometers on a straight track with a grade of $1^\circ 10'$ (see figure on the next page). What is the vertical rise of the train in that distance?

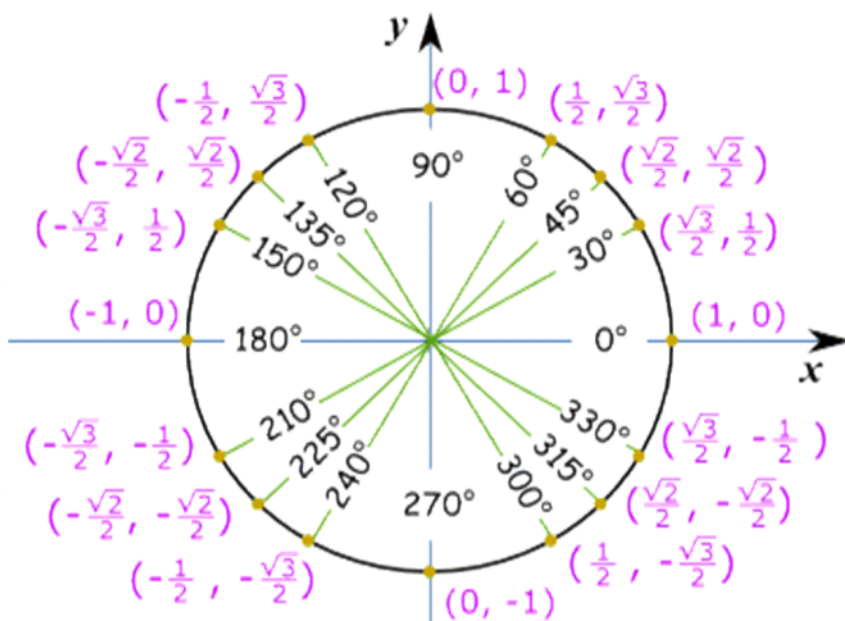


$\sin 1^\circ 10' = \frac{x}{3.5}$

$3.5 (\sin 1^\circ 10') = x$

$x = .0713 \text{ km}$

The Unit Circle



* Alternative method:

47. $\csc \theta = 4$ (a) $\sin \theta$ (b) $\cos \theta$

$\frac{\text{hyp}}{\text{opp}} = \frac{4}{1}$

(c) $\sec \theta$ (d) $\tan \theta$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 1^2 + b^2 &= 4^2 \\ b^2 &= 15 \\ b &= \sqrt{15} \end{aligned}$$

$\frac{opp}{hyp}$ a) $\sin \theta = \frac{1}{4}$ $\frac{adj}{hyp}$ b) $\cos \theta = \frac{\sqrt{15}}{4}$

$\frac{hyp}{adj}$ c) $\sec \theta = \frac{4}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$ $\frac{opp}{adj}$ d) $\tan \theta = \frac{1}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$