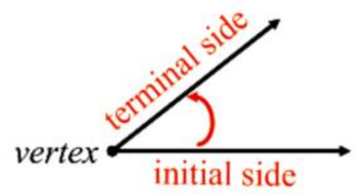
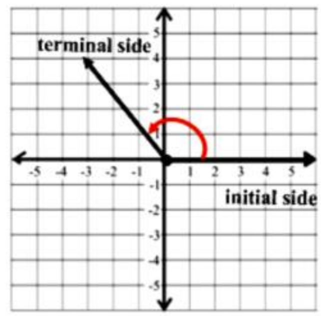
Wednesday, October 31, 2018
6:38 PM

Name KEY Period ________

Precalculus – 4.1 Intro to Unit Circle Notes

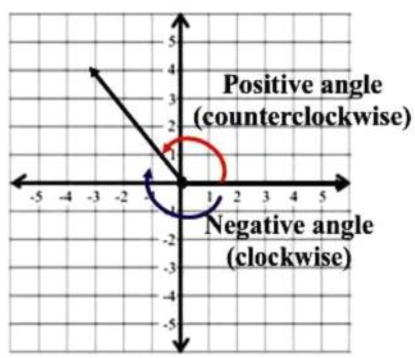
An ***angle*** **i**s determined by rotating a ray about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side**. The endpoint of the ray is the ***vertex*** of the angle. **Note:** It is important to draw an arc with an arrow to indicate the direction of rotation.



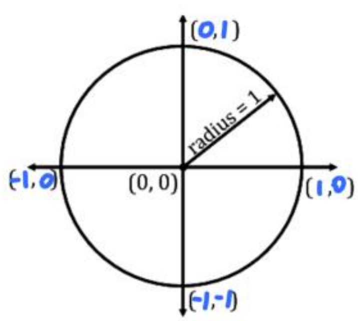


An angle is in **standard position** if the initial side of an angle coincides with the *x*-axis (is on the x-axis).

Positive angles are generated by ***counterclockwise*** rotation, and **negative angles** by ***clockwise*** rotation as indicated by the directional arc and arrow.



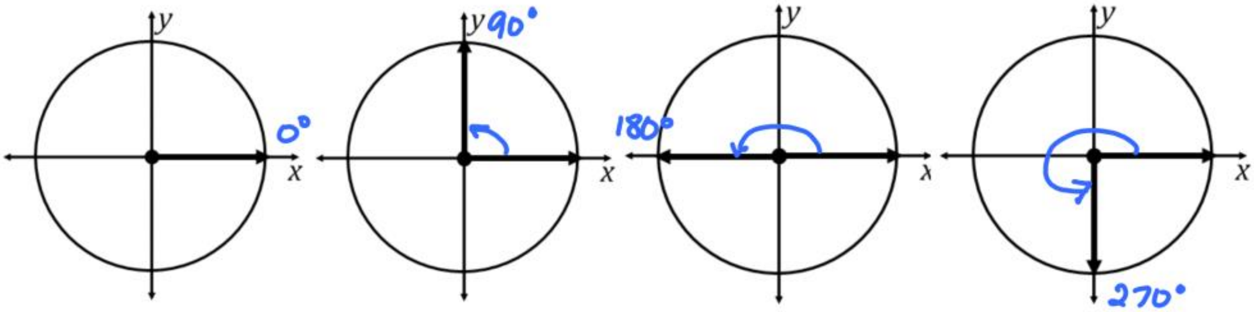
A Unit Circle - A **unit circle** is a circle whose center is at the origin and whose radius is 1 unit.



Ex. 1: Can you name 4 points on the unit circle?

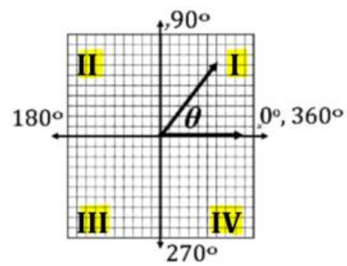
A **Quadrantal Angle** has its terminal side on either the *x* or *y*-axis when drawn in standard position.

Ex. 2: List 5 angle measures for quadrantal angles as you rotate counterclockwise from standard position.



Quadrants – The Greek letter **theta** (θ) is used to represent angles.

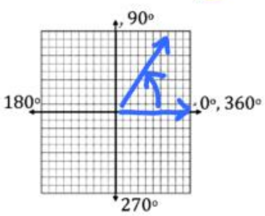
The phrase "the terminal side of θ lies in quadrant…" is often abbreviated by simply saying that "θ lies in quadrant…". Angle θ (pictured to the right) lies in quadrant I.



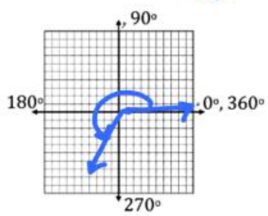
The terminal sides of quadrantal angles $(0°, 90°, 180°, 270°, 360°, \ldots)$ do not lie within quadrants.

Ex. 3: Determine in which quadrant each angle lies by sketching the angle in standard position.

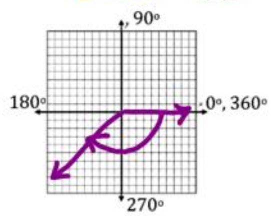
52° I



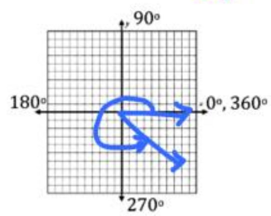
235° III



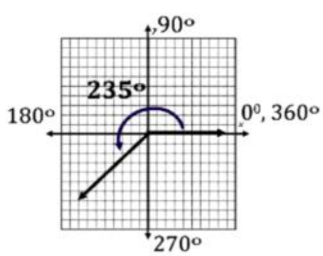
$-140°$ III

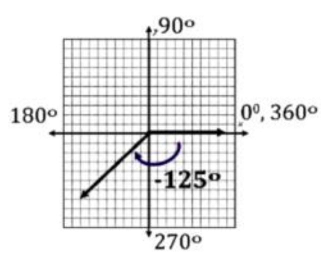


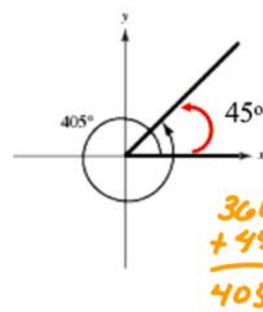
300° IV



Coterminal Angles are angles that have the same initial and terminal sides, but may differ by the direction and number of rotations to get to their final position.







$$\begin{array}{r} 360 \\ +45 \\ \hline 405° \end{array}$$

For example, $235°$ and $-125°$ are coterminal angles. $45°$ and $405°$ are also coterminal angles.

To find **coterminal angles** add or subtract multiples of $360°$ (one revolution), as demonstrated in the following example: *(Note that a given angle θ has infinitely many coterminal angles.)*

For the positive angle $30°$, add or subtract multiples of $360°$ to find coterminal angles.

$30° - 360° = -330°$ $\quad$ $30° + 360° = 390°$ $\quad$ $30° + 360°(2) = 750°$

$30°, -330°, 390°$ *and* $750°$ are all coterminal angles.

Ex. 4: Determine two coterminal angles (one positive and one negative) for each angle.

a) $\theta = 215°$

$215° + 360° = 575°$
$215° - 360° = -145°$

b) $\theta = -75°$

$-75° + 360° = 285°$
$-75° - 360° = -435°$

c) $\theta = 108°$

$108° + 360° = 468°$
$108° - 360° = -252°$

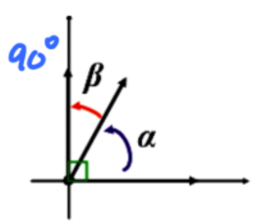
d) $\theta = -410°$

$-410° + 360° = -50°$ ← Negative

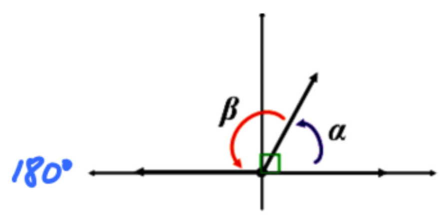
$-410° + (2)360° = 310°$

so ADD another $360°$

Two *positive angles* α (Greek Letter Alpha) and β (Greek Letter Beta) are **complementary** (complements of each other) if their sum is $90°$. Two *positive angles* are **supplementary** (supplements of each other) if their sum is $180°$. It is also possible for an angle to NOT have a complement or supplement (β does not have a complement in the diagram below right).



Complementary Angles



Supplementary Angles

Ex. 5: Find (if possible) the complement and supplement of each angle. If it is not possible, explain why.

a) $3°$

Complement: $90° - 3° = 87°$
Supplement: $180° - 3 = 177°$

b) $64°$

Complement: $90° - 64° = 26°$
Supplement: $180° - 64° = 116°$

c) $130°$

Complement: $90° - 130° = \cancel{-40°}$ none
Supplement: $180° - 130° = 50°$

d) $170°$

Complement: $90° - 170° = \cancel{-80}$ none
Supplement: $180° - 170° = 10°$

Homework: Section 4.1, #31, 34, 35-37, 39-45 odd

In Exercises 31–34, determine the quadrant in which each angle lies.

S **31.** (a) $130°$ (b) $285°$

32. (a) $8.3°$ (b) $257° 30'$

S **33.** (a) $-132° 50'$ (b) $-336°$

34. (a) $-260°$ (b) $-3.4°$

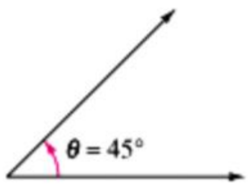
In Exercises 35–38, sketch each angle in standard position.

S **35.** (a) $30°$ (b) $150°$ **36.** (a) $-270°$ (b) $-120°$

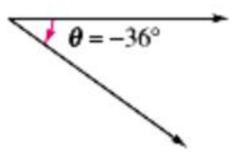
S **37.** (a) $405°$ (b) $480°$ **38.** (a) $-750°$ (b) $-600°$

In Exercises 39–42, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

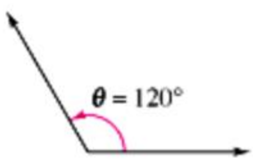
S **39.** (a)



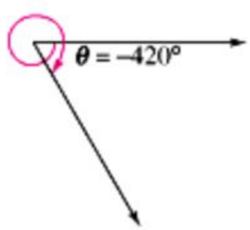
(b)



40. (a)



(b)



S **41.** (a) $\theta = 240°$ (b) $\theta = -180°$

42. (a) $\theta = -420°$ (b) $\theta = 230°$

In Exercises 43–46, find (if possible) the complement and supplement of each angle.

S **43.** (a) $18°$ (b) $115°$ **44.** (a) $3°$ (b) $64°$

S **45.** (a) $79°$ (b) $150°$ **46.** (a) $130°$ (b) $170°$

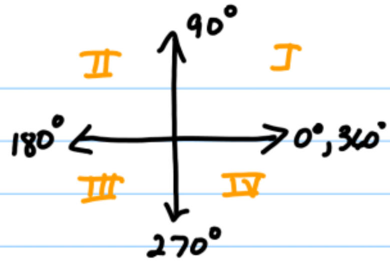
OMIT

In Exercises 47–50, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

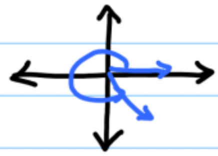
S **~~47~~.** (a) $30°$ (b) $150°$ **~~48~~.** (a) $315°$ (b) $120°$

S **~~49~~.** (a) $-20°$ (b) $-240°$ **~~50~~.** (a) $-270°$ (b) $144°$

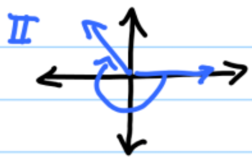
In Exercises 1–34, determine the quadrant in which each angle lies.

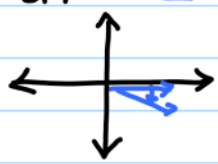


31) a) 130° II b) 285° IV

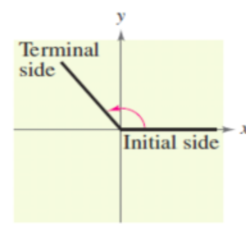


34) a) -260° b) -3,4° IV



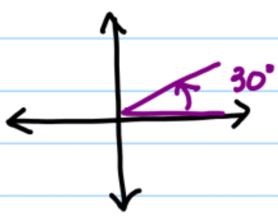


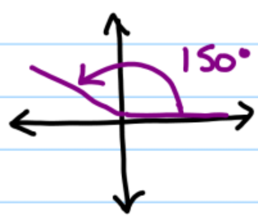
Sketch each angle in standard position.



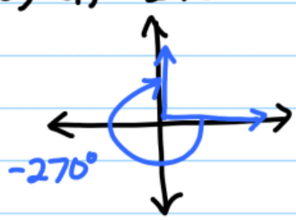
Angle in standard position

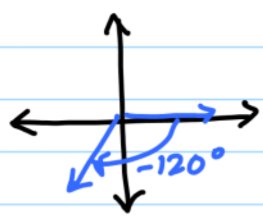
35) a) 30° b) 150°



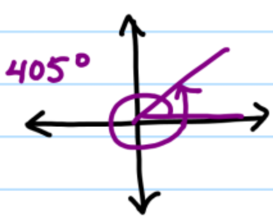


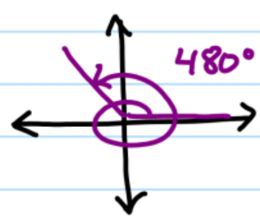
36) a) -270° b) -120°





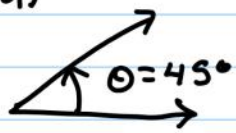
37) a) 405° b) 480°





Determine 2 coterminal angles (one pos. + one neg.) for each angle. Give your answer in degrees.

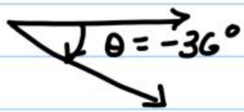
39) a)



$360° + 45° = \boxed{405°}$

$-360° + 45° = \boxed{-315°}$

b)



$360° + -36° = \boxed{324°}$

$-360° + -36° = \boxed{-396°}$

41) a) $\theta = 240°$ $\quad 360° + 240° = \boxed{600°}$

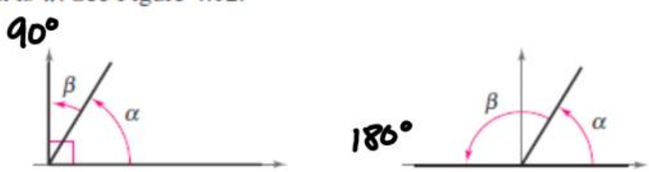
$-360° + 240 = \boxed{-120°}$

b) $\theta = -180°$ $\quad 360° + -180° = \boxed{180°}$

$-360° + -180° = \boxed{-540°}$

Find (if possible) the complement and supplement of each angle.

Two positive angles α and β are **complementary** (complements of each other) if their sum is $\pi/2$. Two positive angles are **supplementary** (supplements of each other) if their sum is π. See Figure 4.12.



Complementary angles

Supplementary angles

43) a) 18° $\quad$ Complement = $90° - 18° = \boxed{72°}$

Supplement = $180° - 18° = \boxed{162°}$

b) 115° $\quad$ Complement = $\boxed{\text{none}}$ $\quad$ 115° is greater than 90°.

Supplement = $180° - 115° = \boxed{65°}$

45) a) 79° $\quad$ Complement = $90° - 79° = \boxed{11°}$

Supplement = $180° - 79° = \boxed{101°}$

b) 150° $\quad$ Complement = $\boxed{\text{none}}$

Supplement = $180° - 150° = \boxed{30°}$