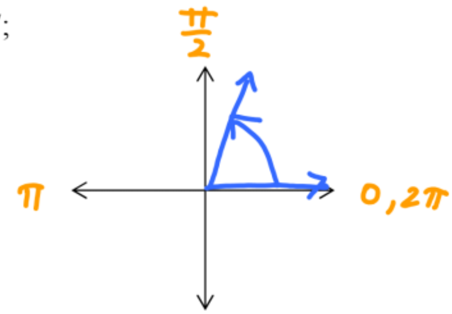
Thursday, November 01, 2018
6:32 PM

KEY

4.1 Day 3 Notes

Hwk: 4.1 #79, 83, 89 (from Day 2), 55 – 77 odd, 87, 93, 107;
VC (skip 8, 9) on SEPARATE piece of paper
Quarter 1 Test ~~Wednesday, 11/15~~ Tues 11/13



Do Now:
Draw $\theta = \frac{5\pi}{12}$ in Standard Position (SP), then find:

a) quadrant I

b) Complement (if possible)

$\frac{6}{6}\cdot\frac{\pi}{2} - \frac{5\pi}{12} = \frac{6\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{\pi}{12}}$

c) Supplement (if possible)

$\frac{12}{12}\cdot\pi - \frac{5\pi}{12} = \frac{12\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{7\pi}{12}}$

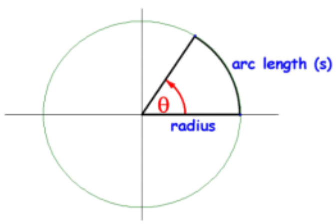
d) Positive coterminal angle

$\frac{5\pi}{12} + 2\pi\cdot\frac{12}{12} = \frac{24\pi}{12} + \frac{5\pi}{12} = \boxed{\frac{29\pi}{12}}$

e) Negative coterminal angle

$\frac{5\pi}{12} - 2\pi = \frac{5\pi}{12} - \frac{24\pi}{12} = \boxed{\frac{-19\pi}{12}}$

New Material:
Central angle: an angle whose vertex is the center of a circle
Radian: measure of a central angle formed when the length of an arc equals length of radius of circle.



$\theta_{radians} = \frac{\text{arclength}}{\text{radius}} = \frac{s}{r}$ or

Think of measuring one radius around circle w/ pipe cleaner
- Formula is in RADIANS!!! Convert from degrees if necessary
- Radian measure has no units (degrees, feet, inches, etc)

Ex. 1) Find the radian measure of the central angle given:
a. Radius (r) = 14 ft, arc length (s) = 8 ft

$\theta = \frac{s}{r}$ $\theta = \frac{8ft}{14ft} = \boxed{\frac{4}{7}}$

b. Radius (r) = 80 mm, arc length (s) = 160 mm

$\theta = \frac{s}{r} = \frac{160\text{mm}}{80\text{ mm}} = \boxed{2}$

Ex. 2) Find arc length (s) on a circle given:

a. r = 4 inches and central angle of $\frac{4\pi}{3}$

$\theta = \frac{s}{r}$ $\quad \frac{4\pi}{3} = \frac{s}{4}$ $\quad \frac{3s}{3} = \frac{16\pi}{3}$ $\quad s = \boxed{\frac{16\pi}{3}}$

b. r = 27 and central angle of 150°

$150° \cdot \frac{\pi \text{ radians}}{180°} = \frac{5\pi}{6}$ $\quad \theta = \frac{s}{r}$ $\quad \frac{5\pi}{6} = \frac{s}{27}$ $\quad 6s = 135\pi$ $\quad s = \frac{135}{6}\pi$

$s = \boxed{\frac{45\pi}{2}}$

* convert to radians

Notice this last angle is given in DEGREES – what should we do? What if the angle isn't a unit circle angle?

Radian measure: another way to measure angles.

$$360^{\circ} = 2\pi \text{ radians} \qquad \pi \text{ radians} = 180^{\circ}$$

$$1^{\circ} = \frac{\pi}{180}\text{radians} \qquad 1 \text{ radian} = \frac{180^{\circ}}{\pi}$$

Conversions Between Degrees and Radians

1. Degrees ➔ Radians: $n^{\circ} \times \frac{\pi \text{ radians}}{180^{\circ}}$
2. Radians ➔ Degrees: $n \text{ radians} \times \frac{180^{\circ}}{\pi \text{ radians}}$

Ex. 3) Convert from Degrees to Radians:

a) 135°

$135° \cdot \frac{\pi \text{ rad}}{180°} = \boxed{\frac{3\pi}{4}}$

b) -36°

$-36° \cdot \frac{\pi \text{ rad}}{180°} = \boxed{-\frac{\pi}{5}}$

Ex. 4) Convert from Radians to Degrees

a) $-\frac{9\pi}{2}$ radians

$-\frac{9\cancel{\pi}}{\cancel{2}} \cdot \frac{180°}{\cancel{\pi \text{ rad}}} = \boxed{-810°}$

b) $\frac{5\pi}{3}$ radians

$\frac{5\cancel{\pi}}{3} \cdot \frac{180°}{\cancel{\pi \text{ rad}}} = \boxed{300°}$

c) 2 radians

$2 \cdot \frac{180°}{\pi \text{ rad}} = 114.59°$

Now we are combining types of problems:
Ex. 5) Find arc length if r = 4 in. and θ = 240°

$$\theta_{\text{radians}} = \frac{\text{arclength}}{\text{radius}} = \frac{s}{r} \text{ or}$$

Convert to radians

$$240° \cdot \frac{\pi \text{ rad}}{180°} = \frac{4\pi}{3}$$

Apply formula:

$$\theta = \frac{s}{r} \qquad \frac{4\pi}{3} = \frac{s}{4\text{in}}$$

$$\frac{3s}{3} = \frac{4\text{in}(4\pi)}{3}$$

$$\boxed{s = 16.76 \text{ in}}$$

Ex. 6) Find arc length if r = 27 in. and θ = 160°

$$\theta_{\text{radians}} = \frac{\text{arclength}}{\text{radius}} = \frac{s}{r} \text{ or}$$

Convert to radians

$$160° \cdot \frac{\pi \text{ rad}}{180°} = \frac{8\pi}{9}$$

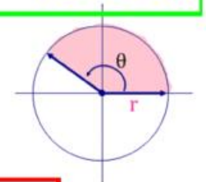
Apply formula:

$$\theta = \frac{s}{r} \qquad \frac{8\pi}{9} = \frac{s}{27\text{in}}$$

$$\frac{9s}{9} = \frac{27\text{in}(8\pi)}{9}$$

$$\boxed{s = 75.40 \text{ in.}}$$

Similarly:
Sector: region of circle bounded by 2 radii and their intercepted arc. a.k.a. wedge or slice of pie



Area of sector:

$A = \frac{1}{2} r^2 \theta$, θ is measure of central angle in <u>radians</u>

Ex. 7) A circle has a radius of 9 in. Find the area of the sector intercepted by a central angle of $\frac{4\pi}{3}$ radians.

$$A = \frac{1}{2} r^2 \theta \qquad A = \frac{1}{2}(9 \text{ in})^2\left(\frac{4\pi}{3}\right) = 54\pi \text{ in}^2 = \boxed{169.65 \text{ sq in}}$$

Ex. 8) Find the area of the sector of a circle with radius of 1.4 miles and a central angle of 330°

Convert to radians

$$330° \cdot \frac{\pi \text{ Rad.}}{180°} = \frac{11\pi}{6}$$

Apply formula: $A = \frac{1}{2} r^2 \theta$

$$A = \frac{1}{2}(1.4 \text{ miles})^2\left(\frac{11\pi}{6}\right) =$$

$$= (.98)\left(\frac{11\pi}{6}\right)$$

$$= \boxed{5.64 \text{ sq. miles}}$$

* use graphing calculator

Lastly, each degree can be broken up into smaller units – **minutes(')** and **seconds(")**

One degree = 60 minutes   One minute = 60 seconds

Ex. 9) Convert each angle measure to decimal degree form. Round to 3 decimal places.

a. 245°10' $\boxed{245.17°}$

① Type in 245
2nd angle ° enter
② Type in 10
2nd angle ' enter

b. -408°16'20" = $\boxed{-408.272°}$

① -408 2nd angle ° enter
② 16 2nd angle ' enter
③ 20 Alpha + enter

Ex. 10) Convert each decimal degree angle measure to DMS (Degree-Minute-Seconds)
Round to seconds if nec.

a. 245.167°

= $\boxed{245° 10' 1.2''}$

b. -408.272°

= $\boxed{-408° 16' 19.2''}$

① type in 245.167
2nd angle DMS enter