Monday, October 22, 2018
5:48 PM

KEY

Precalc 1.9B: Inverse Functions

Obj: To find inverses of functions algebraically & verify algebraically and graphically

Hwk:

1.9B #29, 31, 37, 39, 41, 59, 67, 69, 73, 75; Check answers!

VC on SEPARATE SHEET OF PAPER

1.4 - 1.9 Test Thurs (Per. 5/6 & 9/10)/Fri (Per. 1,);

Scientific calcs only

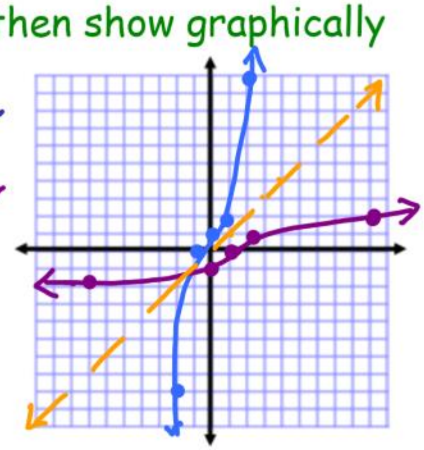
Do Now:

Verify that f & g are inverses algebraically, then show graphically

$f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$

$f(g(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$ ✓

$g(f(x)) = g(x^3+1) = \sqrt[3]{x^3+1-1} = \sqrt[3]{x^3} = x$ ✓



$p(x) = x^3$

x	y	y+1
-2	-8	-7
-1	-1	0
0	0	1
1	1	2
2	8	9

g(x)

x	y
-7	-2
0	-1
1	0
2	1
9	2

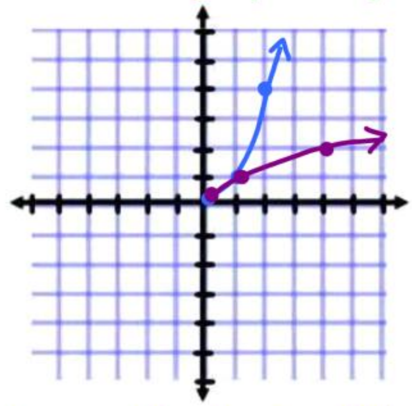
Recap:

Inverse ($f^{-1}(x)$) "undoes" operations

- Swap x and y coordinates
- Domain of original ⇨ range of inverse
- Range of original ⇨ domain of inverse
- To verify, show $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
- To graphically det. if function use VERTICAL LINE TEST
- To det. if inverse function exists, use HORIZONTAL LINE TEST
- A function and its inverse are symmetric wrt y = x (every point is the same distance from the line y = x)

Ex. 1) Verify f & g are inverses algebraically then show graphically

$f(x) = x^2$, $x \geq 0$ $\qquad g(x) = \sqrt{x}$



$f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$ ✓

$g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$ ✗

f(x)

x	y
0	0
1	1
2	4

g(x)

x	y
0	0
1	1
4	2

Note: These are **NOT** inverses - f(x) does not pass the horizontal line test.

BUT if we **RESTRICT THE DOMAIN**, then they ARE inverses!

D_f: ~~$(-\infty, \infty)$~~ $\qquad D_g$: $[0, \infty)$

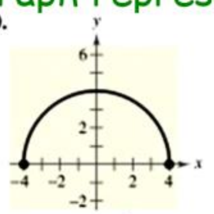
⇨ $[0, \infty)$

One-to-One Functions

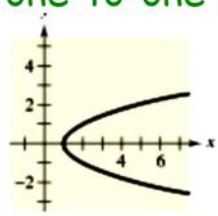
A function is **one-to-one** if each y-value corresponds to **exactly one** x-value. A function has an inverse **IF AND ONLY IF** it is one-to-one. i.e. passes BOTH horizontal & vertical line tests!

Ex. 2) Does the graph represent a one-to-one function?

30.

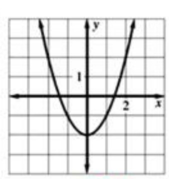


NO



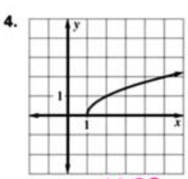
Not a function

3.



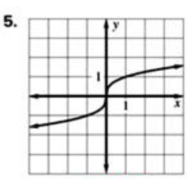
NO

4.



yes

5.



yes

To FIND the Inverse Algebraically:

1. Det. if f has an inverse (horiz. line test)
2. Replace f(x) with y.
3. Switch x and y, then solve for y
4. Replace y with $f^{-1}(x)$. Check for restrictions on domain
5. Verify f and f^{-1} are inverses: show domain & range swapped <u>and</u> $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

***Do NOT confuse <u>VERIFYING</u> with <u>FINDING</u> inverses!**

Ex. 3) Find the inverse:

a. $f(x) = x^3 + 1$ (where have you seen these before, earlier today?)

$y = x^3 + 1$

$x = y^3 + 1$

$x - 1 = y^3$

$\sqrt[3]{x-1} = y$ $\qquad f^{-1}(x) = \sqrt[3]{x-1}$

b. Det. if $f(x) = \dfrac{5-3x}{2}$ has an inverse. Find $f^{-1}(x)$. Verify.

$y = \frac{5}{2} - \frac{3}{2}x$ → linear, passes horizontal line test.

$x = \dfrac{5-3y}{2}$

$2x = 5 - 3y$

$2x - 5 = -3y$

$y = \dfrac{2x-5}{-3}$

$f^{-1}(x) = \dfrac{2x-5}{-3}$

↑

*call this g(x)

verify:

$f(g(x)) = \dfrac{5 \cancel{-3}\left(\frac{2x-5}{\cancel{-3}}\right)}{2}$

$= \dfrac{5 + (2x-5)}{2} = \dfrac{2x}{2} = x$ ✓

$g(f(x)) = \dfrac{\cancel{2}\left(\frac{5-3x}{\cancel{2}}\right) - 5}{-3}$

$= \dfrac{5-3x-5}{-3} = \dfrac{-3x}{-3} = x$ ✓

$f(g(x)) = g(f(x)) = x$ ✓

Ex. 4) Show the domain of f equals the range of f^{-1}, and the range of f is equal to the domain of f^{-1}.

$f(x) = \sqrt{x-2}$ $\qquad$ $f^{-1}(x) = x^2 + 2, x \geq 0$

x	f(x)
2	0
3	1
6	2
11	3

x	$f^{-1}(x)$
0	2
1	3
2	6
3	11

✓

Ex. 5) Find inverse of $f(x) = (x+3)^2$, $x \geq -3$ if it exists. What is the domain and range of each?

$y = (x+3)^2$

$\sqrt{x} = \sqrt{(y+3)^2}$

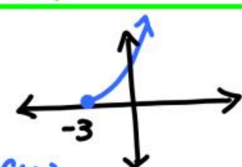
$\sqrt{x} = y+3$

$\sqrt{x} - 3 = y$

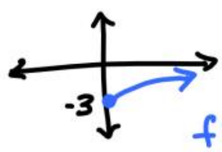
$\boxed{f^{-1}(x) = \sqrt{x} - 3}$

$D_{f(x)}: [-3, \infty)$ $\qquad$ $R_{f(x)}: [0, \infty)$

$D_{f^{-1}(x)}: [0, \infty)$ $\qquad$ $R_{f^{-1}(x)}: [-3, \infty)$



f(x)



$f^{-1}(x)$

Ex. 6) Given $f(x) = \frac{1}{8}x - 3$, find $(f^{-1} \circ f^{-1})(6)$

$(f^{-1} \circ f^{-1})(6) = \frac{1}{8}(6) - 3 = \boxed{-2.25}$

Closure: with your neighbor, discuss

1) What is the dif. between verifying, finding, & showing inverses
2) What does one-to-one mean?