Sunday, October 21, 2018
6:32 PM

KEY

Precalc 1.9A: Inverse Functions

Obj: To verify functions are inverses algebraically & graphically

Hwk: 1.9A #9 - 12, 15, 17, 19, 21, 25, 27

1.4 - 1.9 Test; scientific calcs only, Thurs/Fri

Do Now:

$f(x) = x^2 - 15$ $g(x) = \sqrt{x-1}$ $h(x) = \dfrac{1}{2x-5}$

Find a) $g \circ f$ b) $D_{g \circ f}$ c) $g \cdot h$ d) $D_{g \cdot h}$

a) $g(f(x)) = g(x^2-15) = \sqrt{x^2-15-1} = \boxed{\sqrt{x^2-16}} \leftarrow x^2-16 \geq 0$

$\sqrt{x^2} \geq \sqrt{16}$

$|x| \geq 4$

$-x \geq 4$ OR $x \geq 4$

$x \leq -4$ or $x \geq 4$

b) $D_f: (-\infty, \infty)$, $D_{g \circ f}: \boxed{(-\infty, -4] \cup [4, \infty)}$

c) $g(x) \cdot h(x) = \sqrt{x-1} \cdot \dfrac{1}{2x-5} = \boxed{\dfrac{\sqrt{x-1}}{2x-5}}$

d) $D_g: x-1 \geq 0$, $x \geq 1$ $D_h: 2x-5 \neq 0$ $2x \neq 5$ $x \neq 5/2$ $\boxed{D_{g \cdot h}: [1, 5/2) \cup (5/2, \infty)}$

New Material:

Inverses –an inverse ***"undoes"*** what was already done. You know from solving equations that + ⇨ - , - ⇨ +, x ⇨ ÷, and ÷ ⇨ x. These are **inverse operations**.

The **inverse function $f^{-1}(x)$** is formed by ***switching or swapping*** x & y coordinates in a set of ordered pairs.

Ex. 1) $f(x) = x + 4$: {(1, 5), (2, 6), (3, 7), (4, 8)}

$f^{-1}(x) = x - 4$: {(5, 1), (6, 2), (7, 3), (8, 4)}

**** The domain of f(x) becomes the range of $f^{-1}(x)$.***
The range of f(x) becomes the domain of $f^{-1}(x)$.

When asked to **VERIFY ALGEBRAICALLY** two functions are inverses, you must prove **$f(f^{-1}(x)) = x$ AND $f^{-1}(f(x)) = x$.**

[i.e. $f(g(x)) = g(f(x)) = x$.]

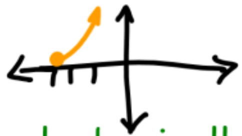
Ex. 2) Verify algebraically that f and g are inverses.

$f(x) = 1 - x^3$ $\qquad$ $g(x) = \sqrt[3]{1 - x}$

$$f(g(x)) = f(\sqrt[3]{1-x})$$
$$= 1 - (\sqrt[3]{1-x})^3$$
$$= 1 - (1 - x)$$
$$= x \checkmark$$

$$g(f(x)) = g(1 - x^3)$$
$$= \sqrt[3]{1 - (1 - x^3)}$$
$$= \sqrt[3]{x^3}$$
$$= x \checkmark$$

Since $f(g(x)) = g(f(x)) = x$, the functions are inverses.



Ex. 3) Verify algebraically that f and g are inverses.

$f(x) = (x+3)^2,\ x \geq -3 \qquad g(x) = \sqrt{x} - 3$

$f(g(x)) = f(\sqrt{x}-3)$
$= (\sqrt{x}-3+3)^2$
$= (\sqrt{x})^2$
$= x$ ✓

$g(f(x)) = g(x+3)^2$
$= \sqrt{(x+3)^2} - 3$
$= x+3-3 \quad = x$ ✓

Why $x \geq -3$?

Ex. 4) <u>Find</u> the inverse, then verify your answer is the inverse of the original.

$f(x) = 2x + 4;$

$y = 2x+4$
$x = 2y+4$
$\frac{x-4}{2} = \frac{2y}{2}$
$y = \frac{1}{2}x - 2$

$\boxed{f^{-1}(x) = \frac{1}{2}x - 2}$

<u>verify</u> show:

$f(g(x)) = x$
$f(\frac{1}{2}x-2) =$
$2(\frac{1}{2}x-2)+4 =$
$x-4+4 =$
x ✓

$g(f(x)) = x$
$g(2x+4) =$
$\frac{1}{2}(2x+4)-2 =$
$x+2-2 =$
x ✓

Definition of inverse function:

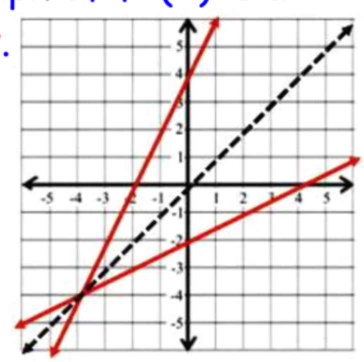
- If $f(g(x)) = x$ and $g(f(x)) = x$ then g is the **inverse function of f**.
- The domain of f must equal the range of g and the range of f must equal the domain of g.

***** f^{-1} will always refer to the inverse function, NOT the reciprocal.***

- If 2 functions are inverses, then the graph of $f^{-1}(x)$ is a **reflection** of the graph in the line **y = x**.

  i.e. if (a, b) is on f,
  then (b, a) is on $f^{-1}(x)$.

Ex: $f(x) = 2x + 4$
$f^{-1}(x) = \frac{1}{2}x - 2$



- Graphically, the **VERTICAL LINE TEST** to determines if it's a **function**. To determine if a relation has an **inverse**, use the **HORIZONTAL LINE TEST**.

Ex. 5) Verify that f & g are inverse functions

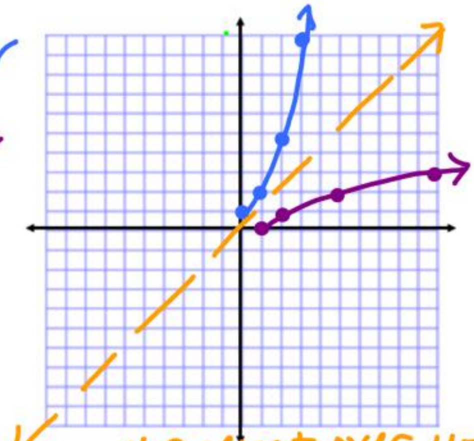
a. algebraically b. graphically

$f(x) = x^2 + 1, x \geq 0$ $g(x) = \sqrt{x - 1}$

a)

$f(g(x)) = (\sqrt{x-1})^2 + 1 = x - 1 + 1 = x$ ✓

$g(f(x)) = \sqrt{x^2+1-1} = \sqrt{x^2} = x$ ✓



b) $f(x) = x^2 + 1$

$g(x) = \sqrt{x-1}$

x	f(x)
0	1
1	2
2	5

x	g(x)
1	0
2	1
5	2

* Reflect over y = x

Ex. 6) Which is the inverse of $f(x) = 7x + 4$?

$\boxed{g(x) = \frac{x-4}{7}}$ or $h(x) = \frac{x-7}{4}$

$y = 7x + 4$

$x = 7y + 4$

$x - 4 = 7y$

$y = \frac{x-4}{7}$

Closure:

<u>Verify algebraically</u> that $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$ are inverses. Then show <u>graphically</u>.

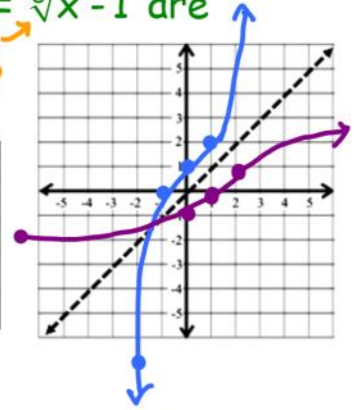
Substitue x values to get

$f(g(x)) = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$ ✓

$g(f(x)) = \sqrt[3]{x^3+1-1} = \sqrt[3]{x^3} = x$ ✓

x	$f(x)$
-2	-7
-1	0
0	1
1	2

x	$g(x)$
-7	-2
0	-1
1	0
2	1



p(x)

x	y
-2	-8
-1	-1
0	0
1	1
2	8

x	f(x)
-2	-7
-1	0
0	1
1	2
2	9