Tuesday, October 16, 2018
5:23 PM

Precalc KEY 1.8B: Composition of functions

Obj: To find the composition of functions and find the domain of such compositions

Hwk: 1.8B #31, 33, 35, 37, 43, 45, 47; + 1.8 C MW

Quiz 1.6 - 1.8 on Thursday 10/18 - No Calculators

Ch. 1 Test (Sect. 1.4 - 1.9) on Thurs (10/25) or FRI (10/26)

Do Now:

If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find

a) $\left(\frac{f}{g}\right)(x)$

$$\frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{4-x^2}} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}} = \boxed{\frac{\sqrt{x(4-x^2)}}{4-x^2}}$$

b) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{4-x^2}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$

$$= \boxed{\frac{\sqrt{x(4-x^2)}}{x}}$$

Find the domain of each. How can you check?

a) $x \geq 0$

$4-x^2>0$

$-x^2>-4$

$x^2<4$

$|x|<2$

$-x<2$; $x<2$

$x>-2$

$-2<x<2$

$\boxed{D: [0, 2)}$

b) $4-x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

$|x| \leq 2$

$-x \leq 2$; $x \leq 2$

$x \geq -2$

$-2 \leq x \leq 2$

$x>0$

$\boxed{D: (0, 2]}$

Recap:

x = input; f(x) = y ← output

You can also use FUNCTIONS or EXPRESSIONS as input into other functions.

Composite functions – functions "composed" or "made up of" 2 functions "nested" together. Start with the INNERMOST function, and substitute the ENTIRE FUNCTION into the OUTERMOST function.

The composition of function f with function g is $(f \circ g)(x) = f(g(x))$

Ex. 1) $f(x) = x + 2$ and $g(x) = 4 - x^2$

a) $(f \circ g)(x) =$

$f(4-x^2) = 4-x^2+2$

$= \boxed{-x^2+6}$

b) $(g \circ f)(x) =$

$g(x+2) = 4-(x+2)^2$

$= 4-(x+2)(x+2)$

$= 4-(x^2+4x+4)$

$= \boxed{-x^2-4x}$

* $(f \circ g)(x)$ is <u>not</u> nec. equal to $(g \circ f)(x)$

c) $(f \circ g)(0)$

$g(0) = 4-(0)^2 = 4$

$f(4) = 4+2 = \boxed{6}$

d) $(g \circ f)(-x)$

$f(-x) = -x+2$

$g(-x+2) = 4-(-x+2)^2$

$= 4-(-x+2)(-x+2)$

$= 4-(x^2-4x+4)$

$= \boxed{-x^2+4x}$

The **domain of composition** $(f \circ g)$ is all x that can be plugged into g and ALSO plugged into f. The domain must work for ALL functions involved.

1. Find the domain of g (innermost function)
2. Find the domain of $(f \circ g)$ (composite function)
3. Give the intersection (most restrictive domain) of the 2

Ex.2) $f(x) = x^2 - 9$, $g(x) = \sqrt{9-x^2}$

Find $(f \circ g)(x)$, D_g, and D_{fog}

$f(\sqrt{9-x^2}) = (\sqrt{9-x^2})^2 - 9 = 9-x^2-9 = \boxed{-x^2}$

D_g: $9-x^2 \geq 0$

$-x^2 \geq -9$

$x^2 \leq 9$

$|x| \leq 3$

$-3 \leq x \leq 3$

$[-3, 3]$

D: $(-\infty, \infty)$

D_{fog}: $\boxed{[-3, 3]}$

Ex. 3) $f(x) = \sqrt{x}$, $g(x) = x^2 + 4$

Find $(f \circ g)(x)$, $(f \circ g)(4)$, D_f, D_g, and $D_{f \circ g}$

$f(g(x)) = f(x^2+4) = \boxed{\sqrt{x^2+4}}$

$g(4) = (4)^2 + 4 = 20$

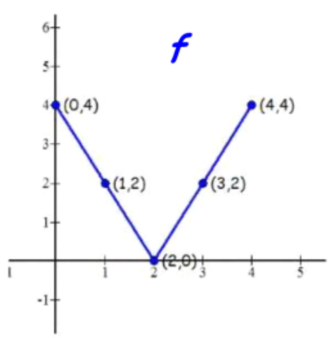
$f(20) = \sqrt{20} = \sqrt{4}\sqrt{5} = \boxed{2\sqrt{5}}$

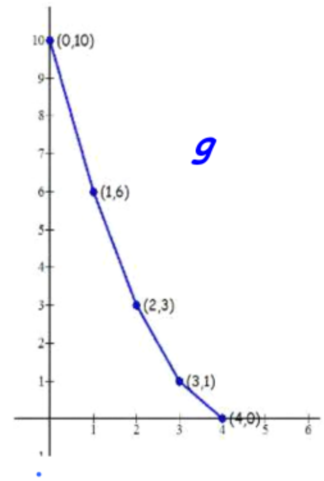
$D_f = [0, \infty)$ $x \geq 0$

$D_g = (-\infty, \infty)$

$D_{f \circ g} = (-\infty, \infty)$

Ex. 4) Use the graphs of f and go to evaluate the functions.





Find:

a) $(f \bullet g)(1)$

$= f(1) \cdot g(1)$

$= (2)(6)$

$= \boxed{12}$

b) $(g \bullet f)(3)$

$= g(3) \cdot f(3)$

$= (1) \cdot (2)$

$= \boxed{2}$

c) $(f+g)(2)$

$= f(2) + g(2)$

$= 0 + 3$

$= \boxed{3}$

d) $(f \circ g)(2)$

$g(2) = 3$

$f(3) = \boxed{2}$

e) $(g \circ f)(3)$

$f(3) = 2$

$g(2) = \boxed{3}$

*Note the dif. in symbols!

Ex. 5) Complete the table:

$f(g(-2))$

x	f(x)	g(x)	f∘g (x)	
-2	5	-1	g(-2) = -1	f(-1) = 4
-1	4	0	g(-1) = 0	f(0) = 3
0	3	1	g(0) = 1	f(1) = 2
1	2	-2	g(1) = -2	f(-2) = 5
2	1	3	g(2) = 3	f(3) = 0
3	0	2	g(3) = 2	f(2) = 1

$$(f \circ g)(x) = f\left(\sqrt{9-x^2}\right)$$
$$= \left(\sqrt{9-x^2}\right)^2 - 9$$
$$= 9 - x^2 - 9$$
$$= -x^2$$

Domain: $f(x) = (-\infty, \infty)$

$$g(x): \begin{array}{l} 9 - x^2 \geq 0 \\ -x^2 \geq -9 \\ x^2 \leq 9 \\ |x| \leq 3 \\ -3 \leq x \leq 3 \\ [-3,\ 3] \end{array}$$

so domain for $f(g(x))$ is $[-3,\ 3]$

$$f + g = \frac{x^2 + 1}{x}; \quad f \cdot g = 1; \quad \frac{f}{g} = x^2;$$
$$D_{f+g}: (-\infty, \infty); \quad D_{f \cdot g}: (-\infty, \infty); \quad D_{f/g}: (-\infty, \infty)$$