Tuesday, October 02, 2018
6:05 PM

Precalc KEY

1.6B Special Functions –
Greatest Int. & Piecewise Functions

Obj: to identify, graph, & evaluate the greatest integer function, step functions, piecewise functions

Hwk: 1.6B #29 – 35 odd, 43 (by hand), 45 & 47 (calc). for #43, 45, 47 find f(2), f(0), f(-1)

Do Now:

1. Identify the parent function. Then match the graph to its equation. Check using a graphing utility.

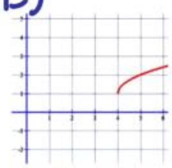
a) $f(x) = 4 + \frac{1}{x+3}$ $g(x) = \frac{1}{x}$

b) $g(x) = \sqrt{(x-4)} + 1$ $f(x) = \sqrt{x}$

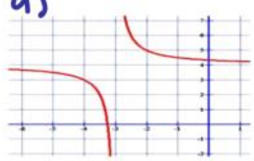
c) $h(x) = 2(x + 3)^3 + 1$ $f(x) = x^3$

d) $j(x) = -x^2 + 8x$ $f(x) = x^2$

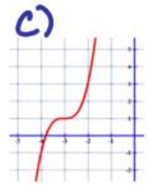
b)



a)



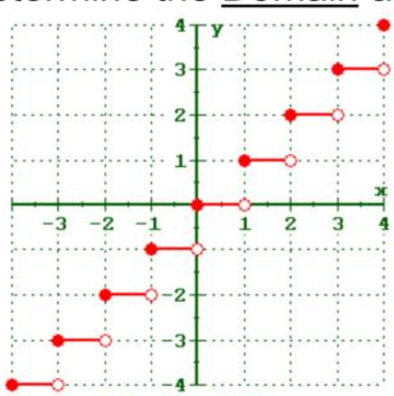
c)



d)

2. Determine the Domain and Range of the following functions.

a.

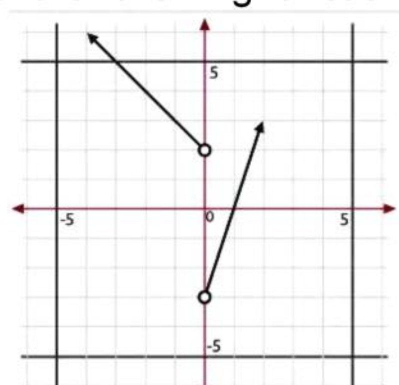


Domain: $[-4, 4]$

Range: $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

All Integers ($\mathbb{Z}$)

b.



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-3, \infty)$

Class Notes:

Greatest Integer Function: see "parent" graph above

- Special piecewise function
- aka step function - graph resembles stairs

$\boldsymbol{f(x)=\text{int}(x)}$ = **greatest integer $\le x$.** Also $f(x)=[\![x]\!]$.

The greatest integer **less than** or **equal to *x***. *(hint: round down.)*

- Major characteristics:
  - Domain: $(-\infty,\infty)$
  - Range: All Integers ($\mathbb{Z}$)
  - y-int: $(0,0)$
  - x-int: all #s in the interval $[0,1)$

Ex. 1)

$[\![2.5]\!]=$ 2 $\quad [\![7.9]\!]=$ 7 $\quad [\![10]\!]=$ 10

$[\![-2.5]\!]=$ −3 $\quad [\![-7.9]\!]=$ −8 $\quad [\![-10]\!]=$ −10

Ex. 2) Given that $f(x)=4[\![x]\!]+7$, find the following…

(a) $f(0)=4[\![0]\!]+7 = \boxed{7}$

(b) $f(6)=4[\![6]\!]+7 = 4(6)+7=\boxed{31}$

(c) $f(6.125)=4[\![6.125]\!]+7 = 4(6)+7=\boxed{31}$

(d) $f(-6.125)$
$=4[\![-6.125]\!]+7$
$=4(-7)+7$
$=\boxed{-21}$

(e) $f(-1.5)=4[\![-1.5]\!]+7 = 4(-2)+7=\boxed{-1}$

(f) $f\left(\frac{5}{3}\right)=4[\![\tfrac{5}{3}]\!]+7 = 4(1)+7=\boxed{11}$

(g) $f(0.999)=4[\![.999]\!]+7 = 4(0)+7=\boxed{7}$

(h) $f(0.009)$
$=4[\![.009]\!]+7$
$=4(0)+7$
$=\boxed{7}$

Ex. 3) Evaluate: $f(x) = [\![x+2]\!]$

$f\left(-\frac{3}{2}\right) =$ $[\![-\frac{3}{2}+2]\!] = [\![.5]\!] = \boxed{0}$

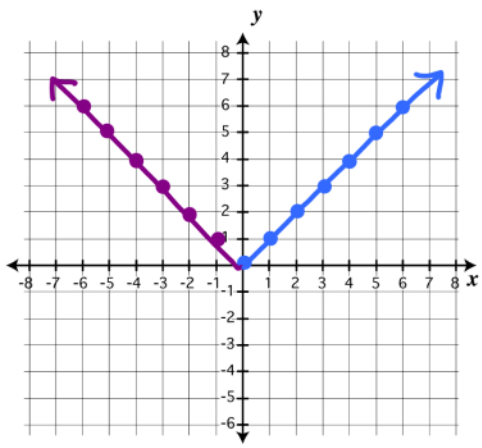
$f(1) =$ $[\![1+2]\!] = \boxed{3}$

$f\left(-\frac{5}{2}\right) =$ $[\![-2.5+2]\!] = [\![-.5]\!] = \boxed{-1}$

PIECEWISE FUNCTIONS

Ex.1) Graph:

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



- What is so special about this function?
- Does it have a special name?

Absolute value function

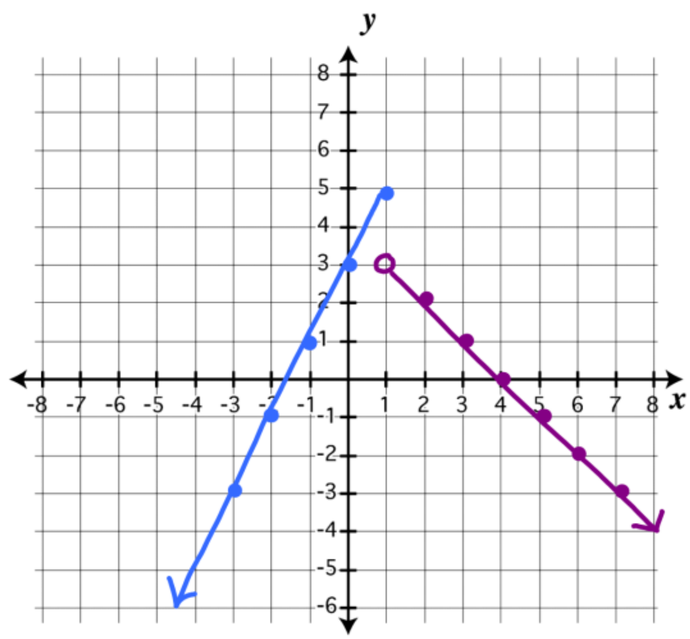
- D: $(-\infty, \infty)$
- Even or odd? Even
- Relative min. at $(0,0)$
- R: $[0, \infty)$
- Decreasing on $(-\infty, 0)$
- Increasing on $(0, \infty)$

Ex. 2) m=2 b=3

$$f(x) = \begin{cases} 2x+3, & x \leq 1 \\ -x+4, & x > 1 \end{cases}$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 5]$



To graph a piecewise function on the calculator…

$$y_1 = (2x+3)/(x \le 1)$$

$$y_2 = (-x+4)/(x > 1)$$

Change to "dot mode"

Ex. 3)

For the following function:

$$f(x) = \begin{cases} |x| + 5 & \text{if } x < -2 \\ 4 & \text{if } -2 \le x < 1 \\ \sqrt{x} + 5 & \text{if } x > 1 \end{cases}$$

a. Determine:

$f(-3)$ $= |-3| + 5 = 8$ $f(-2)$ $= 4$ $f(-1)$ $= 4$

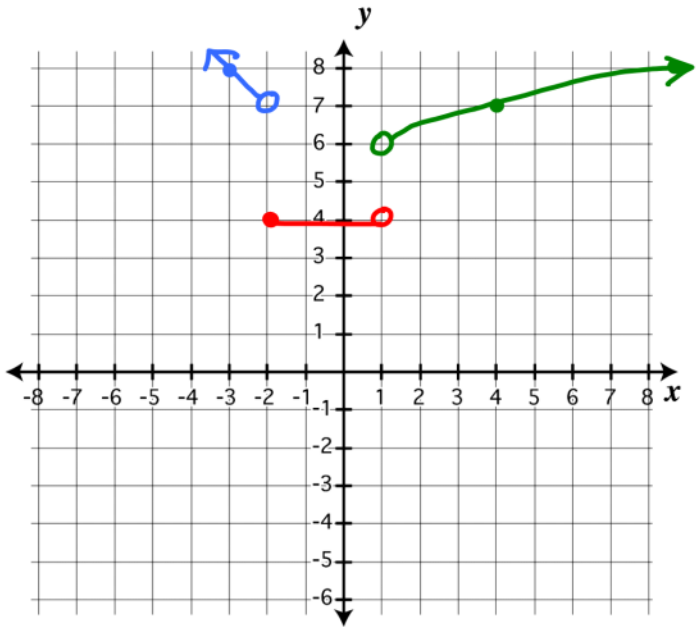
$f(0)$ $= 4$ $f(1)$ undefined $f(4)$ $= \sqrt{4} + 5 = 7$

Check using graph!

b. Determine the Domain.

$(-\infty, 1) \cup (1, \infty)$

c. Graph.



d. Use the graph to determine the Range.

$[4,4] \cup (6, \infty)$

e. Determine all intervals where the function increases, decreases, or is constant.

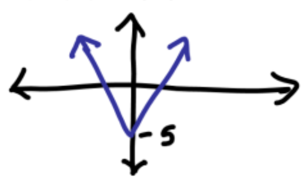
increasing: $(1, \infty)$

decreasing: $(-\infty, -2)$

constant: $[-2, 1)$

Ex. 4) Sketch the graph, then check your prediction using your graphing calculator

a) f(x) = |x| - 5



b) f(x) = x^3 + 4