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HW 1.4C (VC) 79,80, 82-86

VOCABULARY: Fill in the blanks.

1. A relation that assigns to each element x from a set of inputs, or domain, exactly one element y in a set of outputs, or range, is called a function.
2. Functions are commonly represented in four different ways, verbally, numerically, graphically, and algebraically.
3. For an equation that represents y as a function of x, the set of all values taken on by the independent variable x is the domain, and the set of all values taken on by the dependent variable y is the range.
4. The function given by

$$f(x) = \begin{cases} 2x - 1, & x < 0 \\ x^2 + 4, & x \geq 0 \end{cases}$$

is an example of a piece-wise function.

5. If the domain of the function f is not given, then the set of values of the independent variable for which the expression is defined is called the implied domain.
6. In calculus, one of the basic definitions is that of a difference quotient, given by $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$.

Find the difference quotient. Simplify your answer.

79) $f(x) = x^2 - x + 1$, $\frac{f(2+h) - f(2)}{h}$, $h \neq 0$

$$\begin{aligned} f(2+h) &= (2+h)^2 - (2+h) + 1 \\ &= (2+h)(2+h) - 2 - h + 1 \\ &= 4 + 2h + 2h + h^2 - h - 1 \\ &= h^2 + 3h + 3 \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^2 - 2 + 1 \\ &= 4 - 2 + 1 \\ &= 3 \end{aligned}$$

Dif. Quotient $= \frac{h^2 + 3h + 3 - 3}{h} = \frac{h^2 + 3h}{h} = \frac{\cancel{h}(h+3)}{\cancel{h}} = \boxed{h + 3,\ h \neq 0}$

80) $f(x) = 5x - x^2$, $\frac{f(5+h) - f(5)}{h}$, $h \neq 0$

$$\begin{aligned} f(5+h) &= 5(5+h) - (5+h)^2 \\ &= 25 + 5h - (5+h)(5+h) \\ &= 25 + 5h - [25 + 5h + 5h + h^2] \\ &= 25 + 5h - 25 - 10h - h^2 \\ &= -h^2 - 5h \end{aligned}$$

$f(5) = 5(5) - (5)^2$
$= 25 - 25 = \underline{0}$

Diff. Quotient: $\dfrac{-h^2 - 5h - 0}{h} = -\dfrac{h^2 - 5h}{h} = \dfrac{-h(h+5)}{h} = -(h+5)$

$= \boxed{-h-5,\ h \neq 0}$

82) $f(x) = 4x^2 - 2x, \quad \dfrac{f(x+h) - f(x)}{h}, \quad h \neq 0$

$f(x+h) = 4(x+h)^2 - 2(x+h)$ $\qquad f(x) = \underline{4x^2 - 2x}$
$= 4(x+h)(x+h) - 2x - 2h$
$= 4[x^2 + xh + xh + h^2] - 2x - 2h$
$= 4[x^2 + 2xh + h^2] - 2x - 2h$
$= \underline{4x^2 + 8xh + 4h^2 - 2x - 2h}$

Diff. Quotient: $\dfrac{\cancel{4x^2} + 8xh + 4h^2 \cancel{- 2x} - 2h - (\cancel{4x^2} \cancel{- 2x})}{h}$

$= \dfrac{8xh + 4h^2 - 2h}{h} = \dfrac{\cancel{h}(8x + 4h - 2)}{\cancel{h}}$

$= \boxed{8x + 4h - 2,\ h \neq 0}$

83) $g(x) = \dfrac{1}{x^2}, \quad \dfrac{g(x) - g(3)}{x - 3}, \quad x \neq 3$

$g(x) = \underline{\dfrac{1}{x^2}} \qquad g(3) = \dfrac{1}{3^2} = \underline{\dfrac{1}{9}}$

Diff. Quotient: $\dfrac{\frac{9}{9} \cdot \frac{1}{x^2} - \frac{1}{9} \cdot \frac{x^2}{x^2}}{x - 3} = \dfrac{\frac{9}{9x^2} - \frac{x^2}{9x^2}}{x - 3} = \dfrac{\frac{9 - x^2}{9x^2}}{x - 3}$

$= \dfrac{\frac{-(x^2 - 9)}{9x^2}}{\frac{x-3}{1}} = -\dfrac{(x+3)\cancel{(x-3)}}{9x^2} \cdot \dfrac{1}{\cancel{(x-3)}} = \boxed{\begin{array}{c} -\dfrac{(x+3)}{9x^2} \\ x \neq 3, 0 \end{array}}$

84) $f(t) = \frac{1}{t-2}$, $\frac{f(t)-f(1)}{t-1}$, $t \neq 1$

$f(t) = \frac{1}{t-2}$ $f(1) = \frac{1}{1-2} = \frac{1}{-1} = -1$

diff. quotient: $\dfrac{\frac{1}{t-2} - (-1)}{t-1} = \dfrac{\frac{1}{t-2} + \frac{1}{1}\frac{(t-2)}{(t-2)}}{t-1} = \dfrac{\frac{1}{t-2} + \frac{t-2}{t-2}}{t-1}$

$= \dfrac{\frac{t-1}{t-2}}{\frac{t-1}{1}} = \frac{\cancel{t-1}}{t-2} \cdot \frac{1}{\cancel{t-1}} = \boxed{\frac{1}{t-2}, \ t \neq 1, 2}$

85) $f(x) = \sqrt{5x}$, $\frac{f(x)-f(5)}{x-5}$, $x \neq 5$

$f(x) = \sqrt{5x}$ $f(5) = \sqrt{5 \cdot 5} = 5$

diff. quotient: $\boxed{\frac{\sqrt{5x} - 5}{x-5}, \ x \neq 5}$ * can't simplify any further

86) $f(x) = x^{2/3} + 1$, $\frac{f(x)-f(8)}{x-8}$, $x \neq 8$

$f(x) = x^{2/3} + 1$ $f(8) = (8)^{2/3} + 1 = \sqrt[3]{8^2} + 1 = \sqrt[3]{64} + 1$
$= 4 + 1 = 5$

Diff Quotient: $\frac{x^{2/3} + 1 - 5}{x-8} = \boxed{\frac{x^{2/3} - 4}{x-8}, \ x \neq 8}$

* can't simplify any further